SECTION 21

The Probable End

The development of this Universal Physics would now appear to be essentially complete. There are many remaining opportunities for further investigation of various special forms of physical behavior and further implications of this new physics. But, the general overall structure has been developed and that development is validated by its congruence with physical reality.

One other major development has occurred implicitly in the elaboration of this physics. The 20th Century scientific - philosophical point of view of uncertainty, probabilistic reality, relativity and indeterminism has been directly contradicted by this science of "hard", definitive, specific behavior of the physical universe. The philosophical point of view that the questions, "What is truth ?" and "What is real ?" are meaningless questions without answers has been shown to be not only incorrect but quite negative in that it suppresses inquiry and progress that would otherwise take place.

Truth is that which conforms to and describes reality. Reality is that which is, not only matter and energy in their various forms but also: feelings and emotions, ideas and cultures, languages and arts, and so forth. Whether we can know, sense, measure or understand some aspect of reality or not <u>it</u> <u>still</u>, <u>nevertheless</u>, <u>is</u>.

However, one physics issue yet remains. That is: Does the universe go on forever or does it have an end; do the centers-of-oscillation go on forever, go on oscillating, propagating waves, causing electric field, and so on, forever ? For them to do so is at least "uncomfortable" and would appear to be another infinity that hypothesis must avoid because an infinity cannot occur in material reality.

The medium that is propagated by the myriad centers-of-oscillation is, results in, the entire material universe and all that takes place in it, as has been shown throughout the preceding sections of development. Neither the size of that material universe nor the amount of the U-wave propagation that gives rise to it can be infinite. The centers cannot go on forever propagating at the same rate.

It would seem much more reasonable that, once having started, the oscillations then gradually decay in some sense, decay back toward the zero at which they were before the Origin; that they be the gradual depletion of some source, of some supply of medium. It can easily be shown that such a decay in exponential form involves a finite amount of total propagation, not an infinity. (The area from $t = 0$ to $t = \infty$ under the curve of ε^{-t} equals 1 as obtained by integration).

Such behavior is so common in nature. Radioactive decay, for example, exhibits an exponentially decreasing rate of radioactive emission because the rate of emission is proportional to the amount of undecayed nuclei present and that very emission further reduces the number of undecayed nuclei available to support further emission. It would seem quite reasonable that the U-wave propagation from a center be proportional to some quantity related to the center, some amount of something of which the center consists, and that that quantity be progressively reduced by the on-going propagation, which propagation itself would then progressively decrease in consequence.

In fact, the entire concept of the centers continuously propagating waves seems much more sensible and real if there is a source of that propagation, a supply other than a mere empty singularity. Since it appears that such must be the case let us pursue it and see where the development leads.

The Universal Decay

If the wave propagation from a center-of-oscillation, the outward flow of medium in the form of the propagating U-waves, is to be the gradual depleting of some source, then the source itself must consist of, must be a supply of, that same medium. The only possible location for that source is at the center of the center-of-oscillation. The problem is to characterize it, to describe it.

At the center of every center-of-oscillation is a minute *core*, the region of radius δ, as developed in section *19 - A Model for the Universe (9) - Gravitation*. That region, as "viewed" from its outside has the appearance of a sphere of volume $^4/_3 \cdot \pi\delta^3$ and surface area $4\pi\delta^2$ from which U-wave propagation emanates. The "interior" of that *core* could be the source for the center, its "supply" of medium to be slowly propagated away.

Or, the source for the center-of-oscillation could be the sphere centered on the center-of-oscillation and of radius equal to a half or a full wavelength of the center's propagation. And, of course, it could be of some other size.

A choice here is quite easy to make. Equation 19-52 (gravitational slowing), reproduced below, makes clear that the gradation of a center's U-wave field density must be a smooth inverse square variation from distance δ from the center of the center on outward.

$$\Delta c = \frac{\delta^2}{d^2} \cdot c \qquad (19\text{-}52)$$

That is essential if gravity is to behave as it behaves.

The "supply" of medium for a center-of-oscillation must involve medium at an enormously higher concentration than that at which medium is propagated from that center because the propagation has been going on for billions of years and is expected to so continue. Therefore, whatever distance from the center of the center-of-oscillation the "source / supply" region extends is a region of medium density immensely greater than that propagated. Its outer boundary would be a sharp break in the otherwise smooth inverse square variation outside of that region where the propagation is. Thus, if the "supply" of medium for a center occupied a region of radius greater than δ the inverse square behavior could not be smooth up to as close to the center of the center as δ as required by equation 19-52 and gravitation in general.

But, at equation 19-53 it was shown that the value of δ is $4.050,84 \cdot 10^{-35}$ meters, on the order of $3 \cdot 10^{-20}$ of a proton wavelength. Therefore, the source, the "supply" of medium at the center of every center-of-oscillation cannot be a region of radius on the order of a half or whole wavelength. It must be the spherical region of radius δ, the *core* of each center. That is perfectly compatible with the stopping of all incoming propagation at distance δ from the encountered center as found to be the case in the investigation of gravitation and as is stated by equation 19-52, above. Any other configuration would conflict with the operation of gravitation.

The problem of that *core* region has been difficult with the apparent conflict among the necessities that it be:

1 - more than a singularity to support Newton's laws and gravitation;

The core is a non-zero source from which the outgoing waves are focused to produce the operation of Newton's laws and the mass effect. (Detail notes DN 10 - Analysis of Coulomb Focusing Details and Figure DN10-3.) Its radius, δ, determines the operation of gravitation in accordance with equation 19-52.

2 - a pure singularity for a compatible initial instant of the universe;

The Origin had to start from a singularity. If not then it involved an infinity, a "jump", "step" change from nothing to something.

3 - and not a singularity for the sake of the inverse square relationship for the case of the denominator being zero.

Allowing the denominator of $^{medium}/_{r^2}$ to be zero is unacceptable. The inverse square form must be different at (and near) $r = 0$ from its form on outward with the U-wave propagation.

Necessity 1 relates only to the outer surface of the core. It is resolved by the mass focusing source that the core of radius δ provides and equation 19-52, the behavior of gravitational slowing. To resolve the other two necessities requires addressing the interior composition of the core rather than its size.

The nature of the oscillation within the core that produces the oscillation in the core's propagation, in itself an in-place oscillation not a wave propagating on outward, could consist of medium appearing first at the center of the core and then flowing out to the core's boundary before then reversing and flowing back to the core's center. Or, it could consist of an always uniform distribution of medium throughout the volume of the core with the amount varying from zero to a peak amount and then back to zero in some fashion.

Both necessity 2 and 3, above, require the latter mode, that all within the core be total uniformity. Only that is sufficiently pure, simple and uncomplicated. Only that completely eliminates the inverse square problem at

$r = 0$ by ceasing inverse square operation altogether there and replacing it with total uniformity for within distance δ of the $r = 0$ location, the center of the core. And only that leaves the core still as, essentially, a pure singularity in that there is no dimensional or positional variation within it. There can be no dimension, no position, no measurement within a region that is totally uniform and non-particulate throughout. It is "the same place" no matter "where within it one refers to" -- a singular point.

On the other hand, the core interfaces with, and must so interface with, its surroundings. As a point it cannot do so. Interfacing requires a surface and a point has no surface. The core, then, behaves like (and is) a singularity "within" itself and it interfaces with its surroundings via a "surface" which appears to be (and is from the point of view of external to the core) the surface of a sphere of radius δ.

Then, how does that core propagate U-waves ?

Every oscillation that we know in nature exhibits, and the very theory of oscillations in the abstract requires, that the oscillation consist of two aspects storing and exchanging the energy of the oscillation back and forth by means of a "flow". (With one aspect varying in oscillatory fashion then when that aspect decreases there must be some "place" for its energy to go, a place in which it is stored until it reappears in that aspect when it increases again. It cannot completely disappear or be lost because the oscillation would die. That "place" is the oscillation's second aspect and it obviously must vary in a manner related to the first aspect's variation, but with its energy storage in opposite phase.)

A pendulum, for example, oscillates by the motion (flow) of its swinging mass between peak height in the gravitational field (potential energy) at each end of the swing and peak speed of motion (kinetic energy) at the mid-point between the ends of the swing.

Then the Original oscillation at the start of the universe must have so been: a constant (except for the gradual decay) overall amount of medium in an always uniform distribution of the medium throughout the core, the medium being stored or "expressed" in two alternative forms, each oscillating and storing energy in opposite phase to the other.

Such decaying oscillations in natural reality are of the general form (that is, they can be described in mathematical terms) as follows.

(21-1) $$A \cdot \frac{d^2x}{dt^2} + B \cdot \frac{dx}{dt} + C \cdot x = 0$$

[A, B, & C are constants of the particular physical process that the equation describes. "x" is some time-varying quantity of that process.]

For example:

$$M \cdot \frac{d^2s}{dt^2} + F \cdot \frac{ds}{dt} + K \cdot s = 0$$

in which s is distance, M is a mass, F is a constant that reflects the force effect of friction acting according to the speed, and K is a spring constant that reflects the force exerted by the spring according to its displacement from its relaxed position. Each of the three terms in the expression is a force.

This could represent an automobile suspension system for example with M being vehicle mass, F the characteristic of the shock absorbers and K the characteristic of the springs.

or

$$L\cdot\frac{d^2q}{dt^2} + R\cdot\frac{dq}{dt} + \frac{1}{C}\cdot q = 0$$

in which q is electric charge, L is electrical inductance, R is a electrical resistance, and C is electrical capacitance. Each of the three terms in the expression is an electrical potential, a voltage.

This could represent the part of an electrical or electronic device that controls the frequency of a particular tone or oscillation.

Hereafter the following terminology will apply.

υ, $\upsilon(t)$ and υ_c refer to the core medium where υ is "upsilon", the Greek letter u.

u, $u(t)$ and u_c refer, as has already been the usage, to the U-wave propagation.

Lower case as just above is the time-varying quantity. Upper case such as U or Υ is the peak amplitude.

For the particular case of the oscillation of the core of a center-of-oscillation, the corresponding mathematical description / equation would be

$$N\cdot\frac{d^2\upsilon}{dt^2} + O\cdot\frac{d\upsilon}{dt} + \frac{1}{S}\cdot\upsilon = 0 \tag{21-2}$$

[where N, O, & S are treated in development further below]

where the significance and values of N, O, and S remain to be determined.

The solution to all equations of this form is that the dependent variable (υ in equation 21-2) is a function of time t consisting of a sine or cosine form of oscillation multiplied by an exponential decay that damps the oscillation, that is, a form as in equation 21-3, below. (See detail notes DN12.)

$$\upsilon(t) = \Upsilon_c\cdot\varepsilon^{-t/\tau}\cdot[1 - \mathrm{Cos}(2\pi f\cdot t)] \tag{21-3}$$

[τ, Greek "tau", is the constant of the decay.]

If the damping is small (if τ is much greater than the oscillation period, $1/f$) the oscillation goes on for many cycles (desirable in the tone generator but a bad problem in the automobile suspension). If the damping is large the oscillation is snuffed out before even one full cycle (better for the automobile suspension but unusable as a tone generator). The boundary case, which is called "critical damping", is frequently important in real world applications.

The time constant of the decaying exponential damping is

$$\tau = \frac{2\cdot N}{O} \tag{21-4}$$

$$\left[\text{Damping} = \varepsilon^{-t/\tau}\right]$$

and the oscillation is at the frequency

(21-5) $$f = \frac{1}{2\pi}\cdot\left[\frac{1}{N\cdot S} - \frac{1}{\tau^2}\right]^{1/2} \qquad \left[\text{Oscillation} = \text{Cos}[2\pi f\cdot t]\right]$$

Amplitude, phase, a constant level component as the 1 in equation 21-3, and so forth, are determined by the particular actual circumstances of the physical situation that equation 21-2 describes (the initial conditions).

We can expect that the value of τ is quite large. The inverse square effect assumes that there is no decay. If any significant decay could occur during the time that U-waves travel from a source to an encountered center then the magnitude of arriving waves would lack decay that did take place at the encountered center during their transit time. That would yield a different result than would be the case with no such decay. Since the inverse square behavior is soundly and precisely verified the decay that takes place during U-wave transit times must be essentially negligible (that is negligible until very large transit times are considered such as those over astronomical distances).

Later in this section it is developed that τ is on the order of $3.6\cdot10^{17}$ seconds (11 billion years). If τ^2 is large compared to $N\cdot S$ (which is certainly the case for a center-of-oscillation having $\tau > 10^{17}$ seconds and oscillation period, T, on the order of 10^{-21} seconds (electron) or 10^{-24} seconds (proton)) then the $1/\tau^2$ can be omitted from equation 21-5 as relatively too small to have any detectable effect.

(21-6) $$f = \frac{1}{2\pi\cdot\sqrt{N\cdot S}}$$

To further investigate the fundamental behavior and nature of the core of a center-of-oscillation it is helpful to pursue an analogy with a better understood physical process, the type analogies alluded to in equation 21-1, above. The analogy of an electrical circuit is chosen because it is one of the most direct and simple in its behavior and because its electric current (charge flow) is conceptually close to the quantities (medium and its flow) involved in the behavior of a center-of-oscillation and its core.

In order to pursue the analogy it is necessary to first review the pertinent characteristics of the electrical analogue by itself. The electrical analogue of equation 21-1, above, is an electrical circuit consisting of inductance, L, resistance, R, and capacitance, C, connected in series as in Figure 21-1, below. The flow is referred to as *electric current*, i, which is the rate of flow of electric charge. Directly associated with the flow is the potential, e, referred to as *electric potential* or *voltage*.

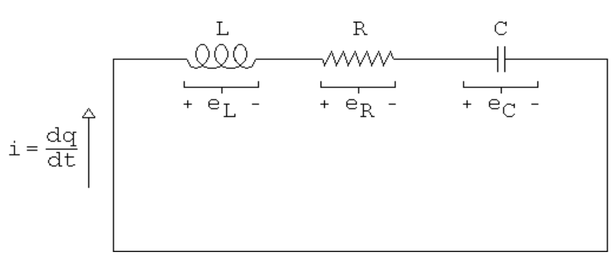


Figure 21-1

Equation 21-7, below, is the behavior of the above series electrical circuit.

(21-7) $e_L + e_R + e_C = 0$ [The sum of the potentials around the circuit must be zero.]

$$L \cdot \frac{d^2q}{dt^2} + R \cdot \frac{dq}{dt} + \frac{1}{C} \cdot q = 0$$

The above analogue was used as the simplest case to initially broach the use of an analogue because the equation describing its behavior is in terms of the flowing quantity, q, analogous to the core's medium, u. However, a different electrical circuit is the more correct analogue for the behavior of the core. That circuit is a parallel one as in Figure 21-2, below.

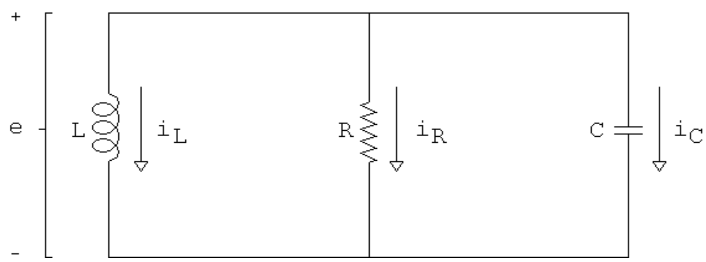


Figure 21-2

Equation 21-8, below, is the behavior of the above parallel circuit.

(21-8) $i_C + i_R + i_L = 0$ [The sum of the currents entering a node must be zero.]

$$C \cdot \frac{de}{dt} + \frac{1}{R} \cdot e + \frac{1}{L} \cdot \int e \cdot dt = 0$$

$$C \cdot \frac{d^2e}{dt^2} + \frac{1}{R} \cdot \frac{de}{dt} + \frac{1}{L} \cdot e = 0 \qquad \text{[The first derivative]}$$

As already presented, the equation describes (the solution to the equation is) an oscillation with amplitude exponentially decaying (if there is initially some current flowing in the inductance or some potential on the capacitance, or both, to supply the initial energy). Energy is stored alternately in the inductance (in its magnetic field in the amount $\frac{1}{2} \cdot L \cdot i_L^2$) and in the capacitance (in its electric field in the amount $\frac{1}{2} \cdot C \cdot e^2$). Some of the energy is lost to the circuit each cycle, it being dissipated as heat energy in the resistance in the amount $R \cdot i_R^2$.

It has already been observed that in the case of a center-of-oscillation the rate of decay is extremely slow relative to the frequency of the oscillation. Consequently it is reasonable to examine the electrical circuit under similar conditions, that is where the value of the resistance, R, is so large that the oscillations decay extremely slowly and appear to us as constant amplitude on-going oscillations. In that circumstance the circuit appears as if the resistance of Figure 21-2, above, were not there, did not exist, and the discussion is of the "steady state" behavior of the circuit rather than its "transient" behavior.

This circuit has the same natural frequency $1/2\pi\cdot[L\cdot C]^{1/2}$. It is a type of circuit that appears commonly in electrical and electronic equipment. In most applications it is not allowed to oscillate at its natural frequency. Rather some external oscillation or combination of oscillations is impressed on the circuit from elsewhere in the overall device by connection at the two nodes.

The relationships of equations 21-7 and 21-8 mean that in the "steady state" the inductance and the capacitance permit only a particular amount of current for a particular voltage present. This controlling factor is called *impedance* it being thought of as impeding the flow of current for an impressed voltage. The steady state impedance relationships for the circuit are

$$e = i\cdot Z$$ ["Z" is the *impedance*.]

$$Z_L = 2\pi\cdot f\cdot L$$

$$Z_C = \frac{1}{2\pi\cdot f\cdot C}$$

[Of course, "f" is the frequency of the impressed voltage.]

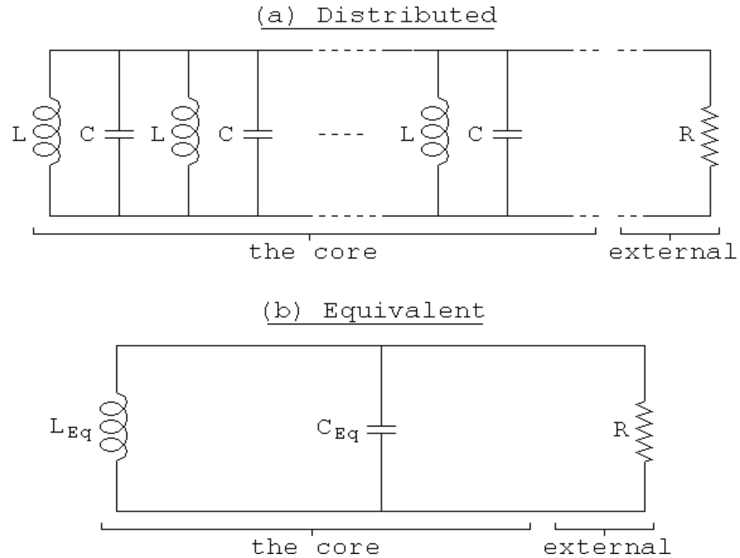
From the above it can be seen that the impedances depend on the frequency involved and behave oppositely: the inductance's impedance increases as frequency increases and that of the capacitance decreases as frequency increases. (The natural, also termed "resonant", frequency is the value of f for which Z_L and Z_C are of equal magnitude.)

If an external electrical source connected to the two nodes of Figure 21-2 were to vary in frequency it would experience different resulting impedances. If its frequency were very large the capacitance would exhibit almost no impedance and would by-pass any effect the large inductive impedance would otherwise have. If its frequency were very small the inductance would exhibit almost no impedance and would by-pass any effect the large capacitive impedance would otherwise have.

The tuning to a selected station or channel on a radio or television is done by making use of this behavior. The selection control causes adjusting of frequencies so that the desired station or channel is centered on the natural frequency of L - C circuits in the radio or television. Then the undesired stations, distant in frequency from the natural frequency, are unable to develop significant voltages and fail to appear.

In continuing to develop the electrical analogue, in the case of the core of a center-of-oscillation there is no externally supplied impressed source. Rather the oscillation is at the natural frequency in a quasi-steady-state condition because the decay is so gradual. The energy is the initial energy corresponding to the Original amount of medium of the center and gradually decaying as medium is lost to the center by propagation outward.

For the center the quantities analogous to the L, C, and R of the electrical circuit are here termed N, S, and O (and named *inertialance*, *storeance* and *opposeance* in analogue to the electrical *inductance*, *capacitance* and *resistance*). Of course these quantities do not occur as single discrete entities as depicted in the above circuit figures. Rather they are myriad, infinitesimal, distributed and intermixed N and S throughout the non-particulate, uniform core of the center. The electrical analogue of that might appear as in Figure 21-3(a), below. However, it is analytically equivalent to the much more simple Figure 21-3(b), especially because, materially, within the core singularity, all is only one uniform, unitary, place.



Electrical Analogue Models of the Core
Figure 21-3

In a circuit such as Figure 21-3(b), above, and taken as having R so large as to be equivalent to omitted (an open circuit), then given an initial current flow and / or an initial potential on the capacitor, there will be continuous oscillation at the frequency $f = {}^1/2\pi\cdot[L\cdot C]^{½}$.

The total amount of charge (analogously, medium) remains unchanged; however the charge oscillates between being stored in the capacitance (analogously storeance) and flowing through the inductance (analogously inertialance). That is accomplished by a sinusoidal flow of current (analogously, flow of medium). The result is a sinusoidal voltage potential (analogously, medium potential) on the capacitance (storeance), lagging the phase of the current (medium flow) by $90°$ and a sinusoidal voltage potential on the inductance (inertialance) leading the current (medium flow) by $90°$. The two voltages (medium potentials) are, then, $180°$ out of phase with each other and sum to zero around the loop.

The potential on the capacitance (storeance) then lags the current through the inductance (inertialance) by $90°$. Since the energies stored are $½\cdot L\cdot i_L^2$ and $½\cdot C\cdot e^2$, the respective energies oscillate at twice the frequency at which the i and e oscillate. Therefore the $90°$ out of phase i and e result in $180°$ out of phase energies. The total energy is constant (less the decay) but is alternately stored in the inductance (inertialance) and the capacitance (storeance).

It is impossible in natural reality for the electrical circuit to have an infinite parallel electrical resistance (that is R not there). All such circuits have at least some finite resistance, have some such effect, whether intended or not. Thus the above circuit must gradually lose energy to that resistance. The energy appears as heat in the electrical circuit. It represents the removal of a small portion of the charge that was participating in the oscillation from further participation (reflected as decreased current and potentials).

Analogously, the opposeance of the center's core removes a minute portion of the medium from participating in the core oscillation. That is the cause of and that is the flow of propagated medium out of the core, it appearing as electric field rather than as heat.

Since, as already presented in section *10 The Probable Beginning*, the oscillation must be of *[1 - Cosine]* form then, in the absence of any decay (opposeance = ∞ as taken above), the medium amount, $\upsilon(t)$, (analogous to charge amount, q) (involved in being stored in the core storeance and involved in flowing in the core inertialance analogously to electric charge in the *L-C-R* analogue) and the medium flow, $j(t)$ (analogous to electric current), are as follows.

$$(21\text{-}9) \qquad \upsilon(t) = \upsilon_c \cdot [1 - \mathrm{Cos}(2\pi \cdot f \cdot t)]$$

and

$$(21\text{-}10) \qquad j(t) = \frac{d[\upsilon(t)]}{dt} = \upsilon_c \cdot 2\pi f \cdot \mathrm{Sin}(2\pi \cdot f \cdot t)$$

These are depicted in Figure 21-4 below.

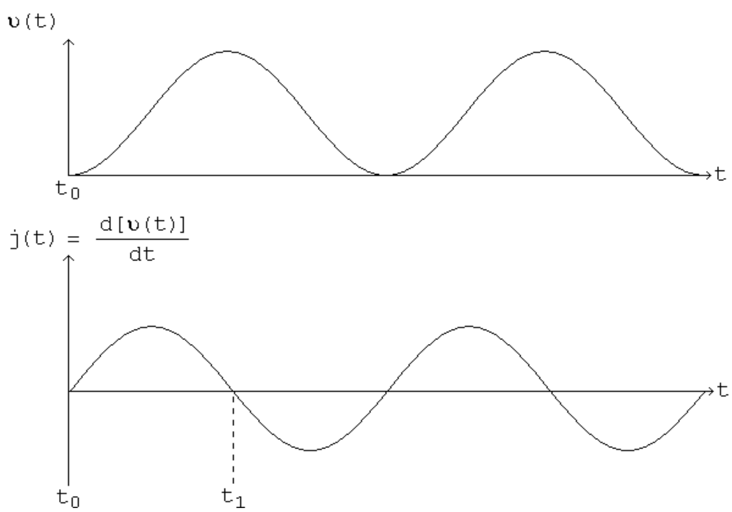


Figure 21-4

This, in itself produces no propagation. Just as a pendulum (if it has no losses due to friction or whatever) oscillates freely and continuously, and certainly without throwing off any of the metal of which it is composed at each cycle, so the above description is of medium flowing between the inertialance and storeance of the core, varying (out of phase) in each between more and less with no propagation, no change in the average amount.

But, it has been determined that this oscillation, this core of the center with its supply of medium, must decay. That means that the υ_c of equations 21-9 and 21-10, above, must gradually decay in magnitude being as the $\upsilon_c(t)$ of equation 21-11, below.

$$(21\text{-}11) \qquad \upsilon_c \Rightarrow \upsilon_c(t) = \upsilon_c \cdot \varepsilon^{-t/\tau}$$

In the graph of Figure 21-4, above, for example at time t_1, when the positive (increasing the medium amount) flow of medium changes to negative flow, the decay (not shown in the figure) would have reduced the amplitudes somewhat. The negative (decreasing the medium amount) flow of medium would be of less amplitude than was the prior medium flow-in. The result ? Propagation of the balance of medium outward. The part of the medium that was flowed-in but not flowed-back by the following (decay-reduced) return flow must leave the core; can only proceed outward. There is nothing else for it to do. That quantity, that balance of medium lost to the oscillation, corresponds to / is that which has everywhere herein been termed the U-wave propagation. Treated as constant it now is found to be decaying with the same decay constant, τ, as $\upsilon_c(t)$.

To determine how much that propagation is the substitution called for in equation 21-11 is made into equation 21-9 yielding the medium, $\upsilon(t)$, corrected to include its decay.

$$(21\text{-}12) \quad \upsilon(t) = \upsilon_c \cdot \varepsilon^{-t/\tau} \cdot [1 - \mathrm{Cos}(2\pi \cdot f \cdot t)]$$

and the flow

$$(21\text{-}13) \quad j(t) = \frac{d[\upsilon(t)]}{dt}$$

$$= \upsilon_c \cdot \varepsilon^{-t/\tau} \cdot 2\pi \cdot f \cdot \mathrm{Sin}(2\pi \cdot f \cdot t) - \cdots$$

$$\cdots - \frac{\upsilon_c}{\tau} \cdot \varepsilon^{-t/\tau} \cdot [1 - \mathrm{Cos}(2\pi \cdot f \cdot t)]$$

$$= \upsilon_c \cdot \varepsilon^{-t/\tau} \cdot 2\pi \cdot f \cdot \mathrm{Sin}(2\pi \cdot f \cdot t) - \frac{1}{\tau} \cdot \upsilon(t)$$

$$= \underbrace{j_{circ}}_{\text{Flow associated with or supporting the decaying center's core medium oscillation.}} + \underbrace{j_{prop}}_{\text{Flow that accounts for the balance of the medium, that propagated.}}$$

The roles of the two flows, j_{circ} within the core and j_{prop}, the propagation out of the core, are illustrated in Figure 21-5, below.

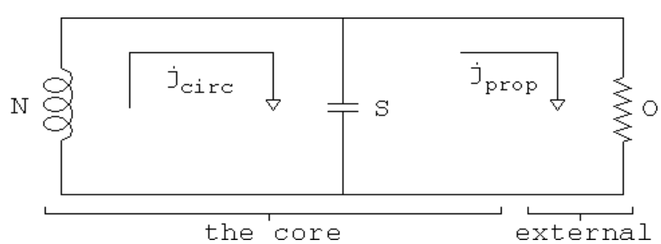


Figure 21-5
Equivalent Model of the Core

The flow, j_{prop}, that represents the propagation of medium outward from the core, which flow is the gradual depletion of the medium amount, $\upsilon(t)$, within the core, that is the core decay, relates to all of the development in the preceding sections of the U-wave propagation from centers-of-oscillation as follows.

(21-14)
$$j_{prop} = \frac{\upsilon_c}{\tau}\cdot\varepsilon^{-t/\tau}\cdot[1 - \mathrm{Cos}(2\pi\cdot f\cdot t)] \qquad \text{[From 21-13]}$$
$$= \underbrace{\frac{\upsilon_c}{\tau}\cdot[1 - \mathrm{Cos}(2\pi\cdot f\cdot t)]}\cdot\underbrace{\varepsilon^{-t/\tau}} \qquad \text{[Rearranged]}$$

is the same thing as, is

$$u(t) = \underbrace{U_c\cdot[1 - \mathrm{Cos}(2\pi\cdot f\cdot t)]}_{\text{u(t) as throughout this work.}}\cdot\underbrace{\varepsilon^{-t/\tau}}_{\text{The u(t) decay.}}$$

U_c is $\upsilon_c/\tau = q$

The U-wave propagation is the inevitable result of the core medium decay. If there were no such decay, there would be no universe as we know it.

The dimensions of the quantities being dealt with need to be clarified here. A full discussion of dimension systems will be found in *Section 3, "Physical Units and Standards*" of *Handbook of Engineering Fundamentals*, First Edition, Ovid W. Eshbach, New York, John Riley & Sons, 1947, as well as other works. Per Eshbach, one could use a different dimension for each physical quantity but it is more economical (as well as more succinctly clear) to use a small set of "fundamental" dimensions with the remainder of the quantities having their dimensions expressed as a combination of the "fundamental" dimensions according to the physical laws (expressed in mathematical relationships) that pertain.

In principal any quantities might be chosen to be the "fundamental" ones; however, practice has been to essentially always make length *[L]* and time *[T]* fundamental. Usually to those is then added mass *[M]*, those three being the common dimensions of mechanics. (It can be observed that these three dimensions seem rather natural and fundamental to we humans.)

Again per Eshbach, a minimum of three fundamental dimensions is sufficient for mechanics but a fourth is needed to treat "heat" and / or "electromagnetism". In heat systems the added fundamental dimension is usually temperature *[Θ]* (because time already uses *"T"*). In treatments of electromagnetism the added fundamental dimension is found to be charge *[Q]* in some cases and permeability *[μ]* in others with several systems not using *[M]* and having two special fundamental dimensions that include one or more of: electric current *[I]*, voltage *[V]*, and resistance *[R]*.

The present analysis and development reduces all phenomena to mechanics, charge for example now being not some esoteric substance or characteristic but the effect of medium flow and oscillation. Only the common three fundamental dimensions *[M]*, *[L]*, and *[T]* are required. Charge, for example, can readily be related to these three dimensions by means of Coulomb's and Newton's laws. This is done in detail notes *DN 3 - The Units of Charge and of Coulomb's Law*.

In brief (using the notation "*{x}*" to mean "the dimensions of *x*"):

(21-15)

a. $\{Force\} = \{Mass\}\cdot\{Acceleration\}$ [Newton's Law]

$$= \frac{M\cdot L}{T^2}$$

b. $\{Force\} = \left\{\frac{Q\cdot Q}{4\pi\cdot r^2}\right\} = \left\{\frac{Q\cdot Q}{L^2}\right\}$ [Natural Form of Coulomb's Law]

c. $$\frac{M\cdot L}{T^2} = \frac{\{Q^2\}}{L^2}$$ [Set a. = b.]

$$\{Q\} = \frac{\sqrt{M\cdot L^3}}{T}$$

d. $\{c\cdot q\} = \{Q\} = \frac{L}{T}\cdot\sqrt{M\cdot L} = \{u\}$

$$\{q\} = \sqrt{M\cdot L}$$

Per equation 21-14 U is related to υ as

(21-16) $U_C = \frac{\upsilon_C}{\tau}$ so that in general $\upsilon(t) = \tau\cdot u(t)$

[υ = core medium amount]
[u = outward flow of core medium]

(21-17) $\{\upsilon\} = \{u\}\cdot\{\tau\}$

$$= \frac{\sqrt{M\cdot L^3}}{T}\cdot T$$

$$= \sqrt{M\cdot L^3}$$

Finishing the conversion of "electromagnetism" quantities being expressed in "mechanics" dimensions:

(21-18)(1) From the speed of light, $\mu_0\cdot\varepsilon_0 = {}^1/_{c^2}$.

$$\{\mu_0\cdot\varepsilon_0\} = \{{}^1/_{c^2}\} = \frac{T^2}{L^2}$$

$$\{\mu_0\cdot\varepsilon_0\} = \frac{T^2}{L^2} = \{\mu\cdot\varepsilon\}$$

(2) From inductive stored energy, $W = ½\cdot L\cdot i^2$.

$$\{W\} = \{½\cdot L\cdot i^2\} = \{½\cdot L\cdot [{}^q/_t]^2\}$$

$$\{W\} = \{Force\cdot Distance\}$$

$$\{W\} = \{Mass\cdot Acceleration\cdot Distance\}$$

$$= M\cdot\frac{L}{T^2}\cdot L = \frac{M\cdot L^2}{T^2}$$

(21-18 continued)

$$\frac{M\cdot L^2}{T^2} = \{L\}\cdot\left[\frac{\sqrt{M\cdot L}}{T}\right]^2 = \{L\}\cdot\frac{M\cdot L}{T^2}$$

$$\{L\} = L$$

(3) From the differential equation of the L-R-C circuit, in which the dimensions of each term must be the same, and aside from the L, R, and C the components are the variables "q" and "t"

$$L\cdot\frac{d^2q}{dt^2} + R\cdot\frac{dq}{dt} + \frac{1}{C}\cdot q = 0$$

$$\left\{L\cdot\frac{d^2q}{dt^2}\right\} = \left\{R\cdot\frac{dq}{dt}\right\} = \left\{\frac{1}{C}\cdot q\right\}$$

$$\{L\}\cdot\frac{\{q\}}{\{t^2\}} = \{R\}\cdot\frac{\{q\}}{\{t\}} = \frac{1}{\{C\}}\cdot\frac{\{q\}}{1}$$

$$\{R\} = \frac{\{L\}}{\{t\}} = \frac{L}{T}$$

$$\{C\} = \frac{\{t\}^2}{\{L\}} = \frac{T^2}{L}$$

(4) From the general formula for capacitance

$$C = \varepsilon\cdot\frac{\text{Surface Area}}{\text{Separation Distance}}$$

$$\{C\} = \left\{\varepsilon\cdot\frac{\text{Surface Area}}{\text{Separation Distance}}\right\}$$

$$\{\varepsilon\} = \left\{C\cdot\frac{\text{Separation Distance}}{\text{Surface Area}}\right\} = \frac{T^2}{L}\cdot\frac{L}{L^2}$$

$$\{\varepsilon\} = \frac{T^2}{L^2} = \{\varepsilon_0\}$$

(5) From (1) and (4) above the dimensions of μ, permeability, are

$\{\mu\} = \{\mu_0\}$ -- (dimensionless)

(μ_0 must be dimensionless in order that $\alpha = ½\cdot\mu_0\cdot c\cdot q^2/_h$, the fine structure constant, be dimensionless as $\{c\cdot q^2\} = \{h\} = M\cdot L^2/_T$.)

The medium of the core of each center oscillates with decay that makes necessary the propagation of medium as waves, as described above. The decays so far developed are as follows.

(21-19)

$$\upsilon(t) = \upsilon_0 \cdot \varepsilon^{-t/\tau}$$ [The medium amount]

$$u(t) = U_0 \cdot \varepsilon^{-t/\tau}$$ [The propagation of medium outward]

$$c \cdot \left[q(t) = q_0 \cdot \varepsilon^{-t/\tau} \right]$$ [The center's charge and and its electric field $q = Q/_c \equiv U/_c = \upsilon/_{\tau \cdot c}$]

With the decaying of these fundamental quantities, the decaying of the core medium, perhaps other quantities related to medium are also decaying such as Planck's constant, h. In fact, with the decay of the rate of propagation of medium a general decay of all medium / wave-related behavior: charge, electric field, gravitation, photon energy, etc. would have to be the case -- the entire universe in gradual exponential decay. And, could it be that, perhaps, c and δ also decay ? Would not a decreasing amount of total core medium perhaps imply a decreasing core "volume" ? And, might not a reduced amount of core medium outward flow per time $(U \equiv Q)$ imply a reduced charge flow at a reduced speed (since $Q = q \cdot c$) ?

Take Planck's constant, h. A medium oscillation has characteristics and behavior to which we apply the terms "energy" and "mass". The amount of energy / mass depends directly on the frequency of the oscillation. Planck's constant is the conversion factor to express the oscillation as energy. (Because, of the two characteristics of any oscillation, amplitude and frequency, the amplitude is fixed at U_c, then the defining characteristic of the oscillation is its frequency. Planck's constant, h, acts on that quantity to give the energy of the oscillation.) It is the energy of a medium oscillation of a particular frequency when the oscillation is of the simple *[1 - Cosine]* form.

If the core medium and propagation of that oscillation are decaying then likely the energy equivalent of that oscillation is decaying. In other words, Planck's constant most likely is decaying. But, what is its rate of decay ? Is it that of $\upsilon(t)$ or what ? The decays of equation 21-19 have dimensions as in Table 21-6, below. Do those decays represent decay of mass (M), of length (L), of time (T) or of some combination of them ?

Quantity	Units	Significance
$\upsilon(t)$	$\sqrt{M \cdot L^3}$	Medium amount
$u(t)$	$\sqrt{M \cdot L^3}/T$	Medium propagation
"	$\sqrt{M \cdot L} \cdot L/T$	Electric field
$c \cdot q(t)$	$\sqrt{M \cdot L} \cdot L/T$	Charge

Table 21-6

Only one, general, overall decay is actually taking place. It is only we human observers who must address the decay in terms of its components. But, consequently, the individual decay rates for each of the components, h, q, etc., must be consistent with each other, that is, when those quantities as decaying variables interact in the various laws of physics (which we express as equations) the resulting decay rates must be consistent. The situation is exactly the same as the essential requirement that the dimensions in which quantities are measured must be overall consistent with each other when those quantities are involved together in physical laws.

Time cannot decay. It is the independent variable. It is only made measurable by the occurrence of events, changes which occur in realized space, the volume dimensions. Time being the independent variable of material reality, whether it decays, varies, or is rigorously constant is beyond our ability to detect in any case. For us it cannot but appear constant.

Mass might be thought to be able to decay, especially in that we "feel" about mass as that it is "substance", substantial, "something to deal with". But mass is merely the ratio of applied force to resulting acceleration. Responsiveness, the more or less inverse of mass, would seem more likely to be a candidate for decay in that it would seem, at first glance, to be medium-dependent. But, responsiveness is a complex interaction that has been shown to vary inversely with frequency, and nothing else, for the cases of simple *[1 - Cosine]* type centers-of-oscillation. As with time, frequency, time's inverse, cannot decay nor "anti-decay" and, therefore, neither can the responsiveness. Certainly, then, mass can not decay nor "anti-decay" as would be its action if responsiveness did decay.

Then the decays of Table 21-6, above, must be decays of the length *(L)* aspect of reality by default. Applying that conclusion to the other fundamental physical quantities Table 21-6 becomes Table 21-7 below. The generic τ of equation 21-19 and before now becomes specific. The flow out of the core at c, in L dimensions of L^1 corresponds to the specific τ and relative decay rate of unity.

Quantity	Units	Significance	Relative Decay Rate	Decay Constant
$\upsilon(t)$	$\sqrt{M\cdot L^3}$	Medium amount	$^3/_2$	$^{2\tau}/_3$
$u(t)$	$\sqrt{M\cdot L^3}/_T$	Medium propagation	$^3/_2$	$^{2\tau}/_3$
$U(t)$	"	Electric field	"	"
$c\cdot q(t)$	"	Charge	"	"
$\delta(t)$	L	Core Radius	1	τ
$c(t)$	$^L/_T$	Speed of travel of medium, light	1	τ
$h(t)$	$M\cdot L^2/_T$	Planck's constant	2	$^\tau/_2$

Decay Rates of Fundamental Physical Quantities
Table 21-7

All of the decays are exponentials with the same base, ε (the natural logarithmic base, not the dielectric constant). In applying these decay rates to the laws of physics, which are expressed in equations involving these quantities, the algebra of exponents applies. In equations with multiplication of variables exponents to the same base are added and for division they are subtracted. Consequently, to find the overall relative decay rate for an expression the relative decay rates of Table 21-7 should be added or subtracted correspondingly. The decay constant is τ times the reciprocal of the relative decay rate.

Taking the decaying quantities and their rates of decay as developed above, the effect on other physical quantities can now be evaluated as in Table 21-8, below.

The Mass-Energy and Oscillation Energy equivalences must have the same decay rates, of course, because they are the same thing; the quantities must be equal at all times: $m \cdot c^2 = h \cdot f$. Likewise, Gravitational Slowing and the Gravitational Constant must have the same decay rates since they produce the same thing, gravitational attraction.

Physical Relationship	Relative Decay Rate
A. Energy:	
Mass-Energy Equivalence	
energy = $m \cdot c^2$	$(0) + (1) \cdot 2 = (2)$
Oscillation Energy Equivalence	
energy = $h \cdot f$	$(2) + (0) = (2)$
B. Force:	
Coulomb's Law	
force = $\dfrac{c^2 \cdot q_1 \cdot q_2}{r^2}$	$(1) \cdot 2 + (1/2) + (1/2) - (1) \cdot 2 = (1)$
Newton's Laws of Motion	
force = $m \cdot a$	$(0) + (1) = (1)$
C. Gravitational Constant	
$G = \dfrac{c^3 \cdot \delta^2}{h}$	$(1) \cdot 3 + (1) \cdot 2 - (2) = (3)$

Table 21-8
Some Major Cosmic Decays

If one attempts to achieve this type of results, the proper correlation of related quantities, using the assumption that the fundamental decay of medium is a decay of mass (M) or is a decay of length (L) and mass (M) jointly the necessary agreements in Table 21-8 do not obtain. Under those conditions different decay rates result for Mass-Energy and Oscillation Energy and different rates for the Coulomb force and the Newtonian force. This tends to confirm the conclusion that the decays are decay of length, (L), only.

That the general process is one involving only length (L) is also consistent with that which takes place in the most broad sense: the realization of space, of volume (L^3), by the propagating medium realizing the otherwise purely latent space, the medium supplying something to occupy, to fill, volume and the medium's oscillatory variation and rate of travel making space mensurable. With the medium decaying its related length and volume effects must correspondingly be decaying.

THE CORE MECHANICS AND THE FUNDAMENTAL CONSTANTS

But, what determined the value of δ? And τ? And, for that matter how did the value of c come to be? Before the Origin none existed. The situation is not like that of sound waves in air where the air is a medium independent of sound waves and the mechanical characteristics of the air determine the speed of sound in it. Before the first core oscillation began and the first U-waves began traveling outward there was no medium, no space, at all. There was nothing independent of the emerging center or external to the propagating waves to determine the magnitude of a length, a time or a speed.

Before the Origin there was not any realized mensurable space at all. Nor were there any changes; time, also, was unmensurable. But, both mensurable time and space would seem to be needed in order to make the speed of the waves or the size of δ or the duration of τ mensurable. It is the waves themselves that cause the realization of the space in which they exist and travel. And δ and τ could be "any size that they chose" since there were no prior things existing that could give relative size to them. It is only comparison of δ and τ to things that followed the Origin that gives us the impression that δ is quite small and τ quite large.

However, those quantities are related to each other by the mechanics of the behavior of the core.

Several quantities that were taken as being of constant amplitude in treatments up to this point are now of varying amplitude as functions of time, c now being $c(t)$ for example. To avoid the cluttering of mathematical expressions with (t) the following convention will now be used:

- upper case letters represent unvarying quantities unless specified with (t) (except Q is $Q(t) = c\cdot q$),
- lower case Latin letters represent functions of time, t (e.g. c is $c(t)$, q is $q(t)$, and so forth),
- lower case Greek letters represent functions of time except for τ and the constants π, α, μ, and ε (natural logarithm base),
  - · δ is a variable $\delta(t)$ and υ is a variable $\upsilon(t)$,
  - · ε (dielectric) is $\varepsilon(t)$ (L of $\{\varepsilon\} = T^2/L^2$ decays).

The overall behavior of the core is a combination of the oscillations treated throughout this work and its gradual exponential decay developed in the present section. The decay is analogous to an envelope of the oscillation. If we ignore the core medium's oscillation for the moment and focus on its decay, only, then the behavior is (from equation 21-3)

$$\upsilon = \upsilon_c \varepsilon^{-t/\tau} \tag{21-20}$$

As already presented, this form of decay occurs quite commonly in nature and in a wide variety of physical processes.

One such process which the core decay resembles is the pumping of gas out of a chamber to create a vacuum. In this case the "gas" is the medium, the chamber is the core, and the pumping is the loss of medium, through the surface boundary of the core, to outward propagation. The process of the pumping, whether of gas out of a vacuum chamber or of medium out of the core is such that:

- The rate of change of the amount of gas (medium) remaining in the chamber (core) and not yet pumped (propagated) is equal to:

    · The density, amount per volume, of the gas (medium) to be pumped out times

    · The pumping speed, that is the volume per time at which the pumping (propagation) occurs.

This is based on the conceptualization of the process as

· The substance to be pumped is uniformly distributed within the chamber,

· a minute increment of volume is then pumped out at the pump during a minute increment of time

· the remaining, un-pumped, part of the substance then automatically, naturally, redistributes itself uniformly within the chamber, and

· the cycle repeats over and over.

From this the rate of change of the amount of medium present within the core would be as follows.

(21-21)
$$\begin{matrix}\text{Rate}\\ \text{of}\\ \text{Change}\end{matrix} = -\left[\begin{matrix}\text{Amount}\\ \text{per}\\ \text{Volume}\end{matrix} \times \left[\begin{matrix}\text{Pumping}\\ \text{Speed}\end{matrix} = \begin{matrix}\text{Surface}\\ \text{of Core}\end{matrix} \times \begin{matrix}\text{Flow}\\ \text{Speed}\end{matrix}\right]\right]$$

$$\frac{d\upsilon}{dt} = -\left[\frac{\upsilon}{{}^4/_3\cdot\pi\cdot\delta^3} \cdot \left[4\cdot\pi\cdot\delta^2\right]\cdot[c]\right] = -\frac{3\cdot c}{\delta}\cdot\upsilon$$

The pumping takes place over the entire surface of the core and the rate at which the outward flow takes place is the speed of medium travel, that which we refer to as the speed of light, c. (Both c and δ are functions of time, also, each decaying as set out in Tables 21-6 and 21-7. However, their decay rates are identical so that their ratio, as in equation 21-21, above, is constant.)

Therefore

(21-22)
$$\frac{d\upsilon}{\upsilon} = -\frac{3\cdot c}{\delta}\cdot dt \qquad \text{[Rearranging equation 21-21]}$$

and, by integration

(21-23)
$$\log_\varepsilon\upsilon = -\frac{3\cdot c}{\delta}\cdot t + C \qquad \text{[C is integration constant.]}$$

$$\upsilon = \upsilon_C\cdot\varepsilon^{-3\cdot c\cdot t/\delta} \qquad [\varepsilon^C \text{ evaluated as } \upsilon_C]$$

Comparing equation 21-23 with 21-20, above, the implied value of the time constant, τ, is as follows.

(21-24)
$$\tau = \frac{\delta}{3 \cdot c}$$

However, that result cannot be correct. Equation 21-24 yields a value of about $4.5 \cdot 10^{-44}$ seconds. It has already been observed that the validity of the inverse square law requires that the value of τ must be quite large whereas the equation 21-24 value is quite minute.

It must be concluded, then, that medium empties from the core at only a minute amount of the volumetric pumping speed used above or, alternatively, that the core volume contains, as medium, an immense supply of volume, of "highly concentrated volume" so to speak. However thought of, it must be from the foregoing that an additional factor that reduces the rate of change of the core medium must be used in equation 21-21 so that it becomes

(21-25)
$$\frac{d\upsilon}{dt} = -\left[\frac{\upsilon}{4/_3 \cdot \pi \cdot \delta^3} \cdot 4 \cdot \pi \cdot \delta^2 \cdot c\right] \cdot \left[\frac{1}{F}\right] = -\frac{3 \cdot c}{\delta \cdot F} \cdot \upsilon$$

where F is the additional factor.

Then,

(21-26)
$$\upsilon = \upsilon_c \cdot \varepsilon^{-3 \cdot c \cdot t/_{\delta \cdot F}}$$

from which

(21-27)
$$\tau = \frac{\delta \cdot F}{3 \cdot c}$$

This result can be obtained specifically rather than being merely inferred as above (again dealing with the peak amplitude not the trigonometric portion of the quantities).

(21-28) a. From the pumping-out-of-a-chamber analysis:

$$j_{prop} = \upsilon \cdot \frac{\text{core surface} \cdot c}{\text{core volume}}$$

$$= Q \cdot \tau \cdot \frac{4 \cdot \pi \cdot \delta^2 \cdot c}{4/_3 \cdot \pi \cdot \delta^3} = Q \cdot \frac{3 \cdot c \cdot \tau}{\delta} \qquad \text{[Here Q should be q]}$$

b. From equation 21-13:

$$j_{prop} = \frac{\upsilon}{\tau} = Q$$

c. Comparing the two above:

$$F \equiv \frac{a.\ j_{prop}}{b.\ j_{prop}} = \left[Q \cdot \frac{3 \cdot c \cdot \tau}{\delta}\right] \cdot \frac{1}{Q} = \frac{3 \cdot c \cdot \tau}{\delta}$$

$$\tau = \frac{\delta \cdot F}{3 \cdot c}$$

Thus the supply of medium that fills the core is in some sense equivalent to some volume, represents, as expressed earlier above, "highly concentrated volume". But, how much volume ? What is the conversion factor from medium to volume, the value of F ?

The precise evaluation of F, t, and d are analytically resolved later in this section. For the moment, it suffices that the universal decay has been confirmed by the discovery of the Pioneer 10 and 11 satellites' "anomalous acceleration" as presented in the scientific paper *physics/9906031* [see the scientific reports archive at URL *http://arxiv.org*]. That confirmation of the decay also validates the value of t, which is

(21-29) $\tau = 11.3373 \cdot 10^{9}$ years.

That result produces for F a value of

(21-30)
$$F = \frac{3 \cdot c \cdot \tau}{\delta} = 7.8 \cdot 10^{60} \quad \text{(dimensionless)}$$

The factor F can be interpreted as

(21-31)
$$F = \left[\frac{3}{\delta}\right] \cdot [c \cdot \tau] = \left[\frac{\text{core surface}}{\text{core volume}}\right] \cdot \left[\begin{array}{c}\text{distance traveled} \\ \text{in time "}\tau\text{" at} \\ \text{speed "c"}\end{array}\right]$$

$$= \frac{\left[\begin{array}{l}\text{volume that would cross the core surface} \\ \text{during time "}\tau\text{" if the rate were constant}\end{array}\right]}{[\text{core volume}]}$$

= number of core volumes that would be propagated during time "ç" if the rate were constant

= total number of core volumes propagated from the beginning to infinity at the actual decaying rate

depicted in Figure 21-9, below.

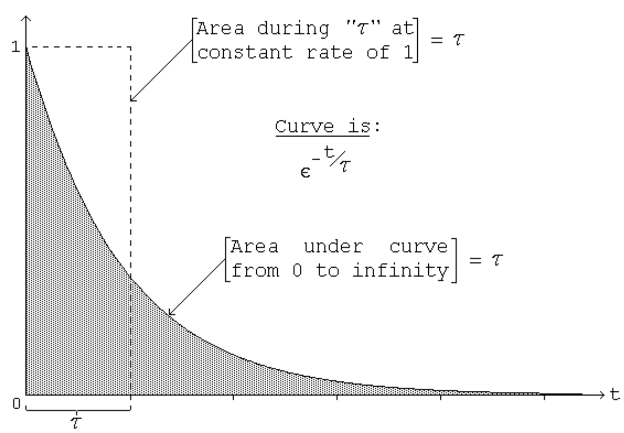


Figure 21-9

Returning to the analogy between the electrical circuit and the core of a center-of-oscillation, the justification for using the analogy is that the two processes are of the same form; each is an exponentially decaying sinusoidal oscillation. They are both described by the same form of differential equation. Therefore the general relationships that result from that differential equation apply to each of the processes. The well known electrical process relationships must have as the actual core behavior their mathematical analogues in the medium process. Table 21-10, below, presents the analogy.

Electrical Circuit			Core-Medium Analogy		
Electric Potential - Charge - Current			Medium Potential - Medium - Flow		
Equation Describing General Behavior					
$C\cdot\frac{d^2e}{dt^2} + \frac{1}{R}\cdot\frac{de}{dt} + \frac{1}{L}\cdot e = 0$			$S\cdot\frac{d^2v}{dt^2} + \frac{1}{O}\cdot\frac{dv}{dt} + \frac{1}{N}\cdot v = 0$		
$\left[\frac{d}{dt}[\Sigma\ \text{currents}] = 0\right]$			$\left[\frac{d}{dt}[\Sigma\ \text{flows}] = 0\right]$		
Set of analogous Quantities					
$\left[\sqrt{M\cdot L}\right]$	charge	q	υ	medium	$\left[\sqrt{M\cdot L^3}\right]$
$\left[\frac{\sqrt{M\cdot L}}{T}\right]$	current	i	j	flow*	$\left[\frac{\sqrt{M\cdot L^3}}{T}\right]$
$\left[\frac{\sqrt{M\cdot L^3}}{T^2}\right]$	electric potential	e	v	medium* potential	$\left[\frac{\sqrt{M\cdot L}}{T^2}\right]$
$\left[\frac{M\cdot L^2}{T^2}\right]$	energy	W	W	energy	$\left[\frac{M\cdot L^2}{T^2}\right]$
$\left[\frac{M\cdot L^2}{T^3}\right]$	power	P	P	power	$\left[\frac{M\cdot L^2}{T^3}\right]$
$[L]$	inductance	L	N	inertialance*	$\left[\frac{1}{L}\right]$
$\left[\frac{L}{T}\right]$	resistance	R	O	opposeance*	$\left[\frac{1}{L\cdot T}\right]$
$\left[\frac{T^2}{L}\right]$	capacitance	C	S	storeance*	$\left[L\cdot T^2\right]$
$\left[\frac{L}{T}\right]$	impedance	Z	Y	retardance*	$\left[\frac{1}{L\cdot T}\right]$

* = new concepts

Table 21-10

The starting point in developing the right (medium) side of the analogy is the already determined circumstance that charge, $c\cdot q$ or Q, is flow of medium, υ, and that electric current is flow of charge, the dimensions in which those quantities must be expressed reflecting their relative time dependencies. Thus medium and its flow appear in somewhat of a "parent" role relative to charge and its flow, current. In a sense, medium is more "real" or "substantial" or "permanent" in that charge is not static anything, it only exists when medium continuously flows. Medium exists when it is.

Given the dimensions of υ and of j, the dimensions of N are obtained from $W = \frac{1}{2} \cdot N \cdot j_N^2$. The dimensions of O and S must then follow according to the dimensions requirements of the differential equation. The dimensions of medium potential then follow from $v_c = O \cdot j_c$. Of course, the dimensions of energy are the dimensions of energy; it must be energy that is stored and alternated in oscillatory fashion.

Table 21-10 (continued)

Electrical Circuit	Core-Medium Analogy
Relationships Among the Various Quantities	
$i = \frac{dq}{dt}$	$j = \frac{d\upsilon}{dt}$
$i_C = C \cdot \frac{de}{dt}$	$j_S = S \cdot \frac{dv}{dt}$
$i_R = \frac{e}{R}$	$j_O = \frac{v}{O}$
$i_L = \frac{1}{L} \cdot \int e \cdot dt$	$j_N = \frac{1}{L} \cdot \int v \cdot dt$
$Z_L = 2\pi \cdot f \cdot L$	$Y_N = 2\pi \cdot f \cdot N$
$Z_C = \frac{1}{2\pi \cdot f \cdot C}$	$Y_S = \frac{1}{2\pi \cdot f \cdot S}$
$W_L = \frac{1}{2} \cdot L \cdot i^2 = \frac{1}{2} \cdot L \cdot \left[\frac{dq}{dt}\right]^2$	$W_N = \frac{1}{2} \cdot N \cdot j^2 = \frac{1}{2} \cdot N \cdot \left[\frac{d\upsilon}{dt}\right]^2$
$W_C = \frac{1}{2} \cdot C \cdot e^2 = \frac{1}{2} \cdot C \cdot \left[\frac{q}{C}\right]^2 = \frac{1}{2} \cdot \frac{q^2}{C}$	$W_S = \frac{1}{2} \cdot S \cdot v^2 = \frac{1}{2} \cdot S \cdot \left[\frac{\upsilon}{S}\right]^2 = \frac{1}{2} \cdot \frac{\upsilon^2}{S}$
$P_R = R \cdot i^2 = R \cdot \left[\frac{dq}{dt}\right]^2$	$P_O = O \cdot j^2 = O \cdot \left[\frac{d\upsilon}{dt}\right]^2$
$\tau = 2 \cdot R \cdot C$	$\tau = 2 \cdot S \cdot O$
$f_{res} = \frac{1}{2\pi \cdot \sqrt{L \cdot C}}$	$f_{res} = \frac{1}{2\pi \cdot \sqrt{N \cdot S}}$

(This *Section 21* is here interrupted in order to present the details of the solution to the differential equation that governs the core oscillation and decay, presented at the beginning of the above Table 21-10.

(The section resumes after detail notes *DN 12 - Solving the 2nd Order Linear Differential Equation with Constant Coefficients*.)

Solving the 2nd Order Linear Differential Equation with Constant Coefficients

If not familiar with differential calculus see detail notes *DN 1 - Differential Calculus, Derivatives* before proceeding here.

The equation to be solved is called the *2nd Order Linear Differential Equation with Constant Coefficients* and is of the general form

(DN12-1) $$A\cdot\frac{d^2x}{dt^2} + B\cdot\frac{dx}{dt} + C\cdot x = 0$$

where A, B, and C are constants reflecting the real physical process that the equation describes and x is a variable that varies with time, t.

The equation is

- *2nd Order* because the highest derivative is the second derivative;
- *Linear* because the derivatives do not have exponents (the exponent-like "2" is the order of the derivative, not a power to which a base is raised);
- *Differential Equation* because it involves derivatives of the variable; and
- *with Constant Coefficients* because none of the derivatives is multiplied by the variable nor an expression involving the variable.

The equation says that "some constant times the second derivative plus some other constant times the first derivative plus a third constant times the variable itself sum to zero". For that to be so, for the terms to be summable at all, the form of the second derivative must be the same as that of the first derivative and both the same as the form of the variable. Otherwise they cannot be summed.

For example the expression

$$3\cdot x + 2\cdot x + x = 6\cdot x$$

can be summed and the sum is clearly $6\cdot x$. However, the expression

$$3\cdot x^3 + 2\cdot x^2 + x = \cdots$$

cannot be summed, that is the terms cannot be combined. Therefore, the solution to the equation must be an expression for x in terms of t, $x = f(t)$, where each of the derivatives of $f(t)$ is of the same form as $f(t)$.

The need for such an expression was encountered and solved in detail notes *DN 2 - Analysis: All Derivatives Finite, Selecting U(t).* The requirement was set out in equation DN2-4, repeated below.

(DN2-4) $$\frac{dU(t)}{dt} = \pm U(t) \qquad \text{[First derivative = } \pm \text{ the function]}$$

The function meeting the requirement is

(DN2-6) $$\varepsilon^t = 1 + t + \frac{t^2}{2\cdot 1} + \frac{t^3}{3\cdot 2\cdot 1} + \cdots$$

$$= 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$$

for which the derivative is

(DN2-7) $$\frac{d[\varepsilon^t]}{dt} = 0 + 1 + \frac{2t}{2} + \frac{3t^2}{3\cdot 2} + \cdots$$

$$= \quad 1 + t + \frac{t^2}{2} + \cdots$$

$$= \varepsilon^t$$

Therefore, the general form of solution to the original problem, equation DN12-1, is

(DN12-2) $$x = \varepsilon^{k\cdot t}$$

where the k is a constant yet to be determined. The procedure is, then to substitute equation DN12-2 into equation DN12-1. The result is as follows.

(DN12-3) 1. Substituting,

$$A\cdot\frac{d^2[\varepsilon^{k\cdot t}]}{dt^2} + B\cdot\frac{d[\varepsilon^{k\cdot t}]}{dt} + C\cdot[\varepsilon^{k\cdot t}] = 0$$

2. Taking the derivatives,

$$A\cdot\left[k^2\cdot[\varepsilon^{k\cdot t}]\right] + B\cdot\left[k[\varepsilon^{k\cdot t}]\right] + C[\varepsilon^{k\cdot t}] = 0$$

3. Dividing through by $[\varepsilon^{k\cdot t}]$,

$$A\cdot k^2 + B\cdot k + C = 0$$

4. Solving for "k" (the quadratic formula).

$$k = \frac{-B \pm \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$$

That makes the exponent, $k \cdot t$, of $\varepsilon^{k \cdot t}$ quite awkward. Furthermore, the solutions of interest, the ones in which there are oscillations, are those for which $4 \cdot A \cdot C > B^2$ so that the square root is the square root of a negative number. That requires using the "imaginary number" symbol, i which is $[-1]^{1/2}$ by definition. A resulting simplified expression for k is then obtained using the following substitutions.

(DN12-4)
$$\alpha \equiv \frac{-B}{2 \cdot A} \qquad \text{[The 1st term of "k"]}$$

$$\beta \equiv \frac{\sqrt{|B^2 - 4 \cdot A \cdot C|}}{2 \cdot A} \qquad \text{[The 2nd term of "k"]} \quad [|\cdots| = \text{magnitude of } \cdots]$$

$$k \equiv \alpha \pm i \cdot \beta \qquad [i \equiv \sqrt{[-1]}\,]$$

Before proceeding further it is necessary to develop two other relationships that will be needed here. These have to do with the expression of the Sine and Cosine functions as infinite series. First their derivatives must be obtained. Referring to detail notes *DN 1 - Differential Calculus, Derivatives*, equation DN1-4 both defines the derivative of an expression and shows how to evaluate it. Paraphrased it is

(DN1-4)
$$\frac{d f(x)}{dx} = \left[\text{Limit as } \Delta x \to 0\right] \text{ of } \left[\frac{f(x+\Delta x) - f(x)}{\Delta x}\right]$$

The derivative of the Cosine is then

(DN1-5)
$$\frac{d[\mathrm{Cos}(x)]}{dx} = \left[\text{Limit as } \Delta x \to 0\right] \text{ of } \left[\frac{\mathrm{Cos}(x+\Delta x) - \mathrm{Cos}(x)}{\Delta x}\right]$$

$$= \left[\text{Limit as } \Delta x \to 0\right] \left[\frac{-2 \cdot \mathrm{Sin}\left[\tfrac{1}{2} \cdot ([x+\Delta x]+x)\right] \cdot \mathrm{Sin}\left[\tfrac{1}{2} \cdot ([x+Dx]-x)\right]}{\Delta x}\right]$$

$\Uparrow$ [A trigonometric Identity]

$$= \;\text{"}\; \left[\frac{-2 \cdot \mathrm{Sin}\left[\tfrac{1}{2} \cdot (2 \cdot x+\Delta x)\right] \cdot \mathrm{Sin}\left[\tfrac{1}{2} \cdot (\Delta x)\right]}{\Delta x}\right] \Leftarrow \text{[Simplify]}$$

$$= \;\text{"}\; \left[\frac{-2 \cdot \mathrm{Sin}\left[\tfrac{1}{2} \cdot (2 \cdot x+\Delta x)\right] \cdot \left[\tfrac{1}{2} \cdot (\Delta x)\right]}{\Delta x}\right] \qquad \text{[Sin(x) = x when "x" is very small as is } \Delta x.]$$

$$= \;\text{"}\; \left[-2 \cdot \mathrm{Sin}[\tfrac{1}{2} \cdot (2 \cdot x+\Delta x)] \cdot [\tfrac{1}{2}]\right] \qquad [\text{Simplify \& } \Delta x \to 0]$$

$$\therefore \quad \frac{d[\mathrm{Cos}(x)]}{dx} = -\mathrm{Sin}(x)$$

By exactly analogous procedure one can obtain

(DN12-6)
$$\frac{d[\mathrm{Sin}(x)]}{dx} = \mathrm{Cos}(x)$$

These results can be used to obtain an expansion of the Sine and the Cosine into a series just as for the exponential, ε^t, as in equation DN2-6, above. The formal method of doing that is to employ Taylor's Theorem in its simplest form, called MacLaurin's Form which states that

(DN12-7)
$$f(x) = f(0) + \frac{f'(0)}{1!}\cdot x + \frac{f''(0)}{2!}\cdot x^2 + \cdots$$

where f' is the 1st derivative,
f" the 2nd derivative and so on.

However, the series can be obtained from reasoning similar to that which was used in obtaining the exponential.

In the exponential case the formulation sought was one that is always its own derivative. Now, from examination of the above equations DN12-5 and DN12-6, the effect of taking repeated derivatives of a sine is as follows.

Which Derivative	Its Form
None	Sin(x)
First	Cos(x)
Second	-Sin(x)
Third	-Cos(x)
Fourth	Sin(x)
Fifth	Cos(x)
Sixth	-Sin(x)

The cycle repeating endlessly
every four orders of derivative.

Such a formulation naturally occurs by constructing a series from every other term of the exponential series and alternating the algebraic signs of the terms, as follows.

The Exponential

(DN2-6)
$$\varepsilon^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$$

The Sine

(DN12-8)
$$\mathrm{Sin}(t) = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \cdots$$

The Cosine

(DN12-9)
$$\mathrm{Cos}(t) = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \cdots$$

Examination of these forms makes clear that the Cosine per equation DN12-9 is the derivative of the Sine per equation DN12-8, that the derivative of that Cosine produces the negative of that Sine, and so on.

The useful aspect of these series is that their relationship to each other and to the exponential means that the trigonometric functions, the Sine and the Cosine can be expressed as algebraic functions of exponentials, as follows.

(DN12-10)
$$\mathrm{Cos}(t) = \frac{1}{2\cdot i}\cdot\left[\varepsilon^{i\cdot t} + \varepsilon^{-i\cdot t}\right]$$
$$[i \equiv \sqrt{[-1]}\,]$$
$$\mathrm{Sin}(t) = \frac{1}{2\cdot i}\cdot\left[\varepsilon^{i\cdot t} - \varepsilon^{-i\cdot t}\right]$$

These relationships can be verified by expanding the expressions, substituting $i\cdot t$ and $-i\cdot t$ for t in ε^{t} per equation DN2-6, above producing equations DN12-8 and DN12-9, above.

The form of the general solution to the original differential equation is then equation DN12-2 with the k of equation DN12-4 substituted.

(DN12-11) $$x = J\cdot\varepsilon^{k\cdot t} = J\cdot\varepsilon^{[\alpha \pm i\cdot\beta]\cdot t}$$

where "J" is a constant to be evaluated for each particular set of conditions.

That solution then develops as follows.

(DN12-12)
$$x = J\cdot\varepsilon^{[\alpha + i\cdot\beta]\cdot t} + J\cdot\varepsilon^{[\alpha - i\cdot\beta]\cdot t}$$
$$= J\cdot\left[\varepsilon^{\alpha\cdot t}\right]\cdot\left[\varepsilon^{i\cdot\beta\cdot t}\right] + J\cdot\left[\varepsilon^{\alpha\cdot t}\right]\cdot\left[\varepsilon^{-i\cdot\beta\cdot t}\right]$$
$$= \varepsilon^{\alpha\cdot t}\left[\underbrace{J\cdot[\varepsilon^{i\cdot\beta\cdot t} \underset{\uparrow}{+} \varepsilon^{-i\cdot\beta\cdot t}]}\right]$$

Per equation DN12-10 this expression is equal to $\mathrm{Cos}(\beta\cdot t)$ if we replace J with $K_1 = {}^{J}/[2\cdot i]$.

The resulting final form of the solution is an exponentially decaying (because α is negative, see equation DN12-4) sinusoidal oscillation at angular frequency β, or cyclic frequency ${}^{\beta}/2\pi$.

(DN12-13) $$x = K_1\cdot\varepsilon^{\alpha\cdot t}\cdot\mathrm{Cos}(\beta\cdot t)$$

There are actually two such solutions. The other one is obtained by using a "-" sign in equation DN12-12 in place of the "+" sign marked by an arrow. It yields a Sine per equation Dn12-10 instead of a Cosine.

(DN12-14) $$x = K_2\cdot\varepsilon^{\alpha\cdot t}\cdot\mathrm{Sin}(\beta\cdot t)$$

The complete solution is the sum of the two solutions since if either one is a solution then the sum is also a solution and that sum is more inclusive.

Evaluation of constants depends upon the actual conditions that the differential equation is treating. In the case of the Origin the initial conditions were evaluated and set in section *10 - The Probable Beginning*. The resulting particular form of the above general solution that describes the Origin is equation 21-12, reproduced, below.

(21-12) $$\upsilon(t) = \upsilon_c\cdot\varepsilon^{-t/\tau}\cdot[1 - \mathrm{Cos}(2\pi\cdot f\cdot t)]$$

For the Origin: $A = S$, $B = {}^{1}/_{O}$, and $C = {}^{1}/_{N}$.