

SECTION 19

A Model for the Universe (9) -- Gravitation

Well ! After all of that it's about time. Here gravity is the most obvious, omnipresent and powerful force in everyday life and yet it has barely even been mentioned up to this point in the discussion. It seems to be such a contradiction that electric field, magnetic field, electromagnetic radiation and so on have been treated in great detail when one seems to be hardly aware of them in daily life; then, now that the development would seem to have been pursued to the point where there is no room for a further major concept, we come to dealing with gravity, which is so significant in daily life.

The reason for this contradiction is as follows. Gravity is actually a very weak force. Relative to the electrostatic force between two protons being (solely for illustration) on the order of magnitude 1 , the gravitational force between them calculates out to on the order of magnitude $0.000\dots(35 \text{ zeros total})\dots0001$. Furthermore, as will shortly be seen, gravity is almost an accident; it is a very minor anomaly in the mechanics already presented. That explains why the subject of gravity has not been taken up before now. But the reason that gravity is so powerful even though weak, the reason for its large role in space and daily life is a fundamental difference in the action of gravity as compared to the other forces.

Electric charge involves positive and negative charges. The electrostatic forces are repulsive or attractive depending upon whether the charges involved are of the same sign or of opposite signs. Likewise, magnetic field involves north and south poles. Magnetic effects involve actions in two opposite directions depending upon the magnetic polarities involved. Generally speaking the universe is overall and locally (down to the atomic level) electrostatically and magnetically neutral. The potentially large forces that could result from the strong electric and magnetic effects are essentially in $+/-$, $\text{attractive/repulsive}$, north/south balance.

But gravitation is always a force of attraction. There is no opposite, compensating, force to offset gravitational attraction with a repulsion. Weak as individual gravitational forces are at the atomic level, they accumulate atom by atom un-offset by any counterbalancing gravitational repulsion. Consequently, the gravitational effects of significant amounts of matter are quite large. (In fact, that being the situation, we are fortunate that the individual gravitational force is so weak. Otherwise the entire universe would have collapsed upon itself long ago.)

What is gravity ? Traditional 20th Century physics does not have the faintest idea. (It hypothesizes "gravitons" and "gravity waves" but has never detected them and cannot incorporate them into an overall theory.) The behavior of gravity is well known in direction, magnitude and so on. What gravity does

and how much it does it is no problem in traditional 20th Century physics. But, there is not any idea at all as to how, why, by what mechanism gravity occurs and operates. This situation is so severe that the "Universal Gravitational Constant", the G in Newton's law of gravitation

$$(19-1) \quad \text{Force of Gravity} = G \cdot \frac{\text{Product of the Two Attracting Masses}}{\text{Separation Distance}^2}$$

is simply a number, unconnected to the rest of physics. 20th Century physics must use the demurrer, "The universal gravitational constant is not, and cannot be in our present state of knowledge, expressed in terms of other fundamental constants."

This is a grave deficiency. We can control nature, and therefore use it, only to the extent that we understand it. We have electric lights, motors, appliances and machinery because we understand how and why electric and magnetic fields behave. We have electronics, radio, TV, communications, and computers because we understand how and why E-M radiation behaves. Our antibiotics, anesthesia, immunization, effective agriculture, artificial fabrics, plastics, etc., are because we understand how and why molecules, compounds, chemical and biological systems behave. We need such an understanding of gravity.

STUDY OF THE PROBLEM AND EXAMINATION OF HYPOTHESES

Just how much does the gravitational force differ from the Coulomb force? Taking the case of two protons separated by one meter distance the calculation (in Standard International, SI, units) is as follows. (The data is from the previously referenced CODATA bulletin.)

· The Gravitational Force of Two Protons at One Meter

$$\begin{aligned} F_g &= G \cdot \frac{m_1 \cdot m_2}{d^2} \\ &= (6.67259 \cdot 10^{-11}) \cdot \frac{(1.6726231 \cdot 10^{-27})^2}{1^2} \\ &= 1.86677 \cdot 10^{-64} \text{ joules/} \text{meter} \end{aligned}$$

· The Coulomb Force of Two Protons at One Meter

$$\begin{aligned} F_c &= c^2 \cdot \frac{q_1 \cdot q_2}{10^7 \cdot d^2} \\ &= (299,792,458)^2 \cdot \frac{(1.60217733 \cdot 10^{-19})^2}{10^7 \cdot 1^2} \\ &= 2.30707956 \cdot 10^{-28} \text{ joules/} \text{meter} \end{aligned}$$

· The ratio of these is

$$\frac{F_g}{F_c} = \frac{1.86677 \cdot 10^{-64}}{2.30707956 \cdot 10^{-28}} = 0.809148 \cdot 10^{-36}$$

and gravitation is seen to be quite small relative to the Coulomb effect.

The principal behavioral characteristics of gravitation can be listed as follows.

- The force is always one of attraction.
- The force is directly proportional to each of the two masses involved in the mutual gravitational attraction (proportional to their product) and is inversely proportional to the square of the distance between them (between their centers of mass).
- The magnitude of the gravitational force is extremely small compared to the usual electrostatic forces.

Following the same line of reasoning as pursued with regard to electrostatic field in section *II - A Model for the Universe (I) - Electric Field and Charge*, the gravitational effect must be due to something that is propagated radially outward from the gravitating mass. The available options are for the effect to be due either to:

- a new propagation not yet treated in this work, or
- an as yet undeveloped aspect or effect of the behavior of centers-of-oscillation and their propagated U-waves.

A new propagation would appear to be a major complication with the need to account for its origin, cause, behavior, lack of effect on the propagation already presented and its effects, and so forth. Such a second, new propagation would be quite contrary to the essential simplicity of nature. It would seem that a previously undeveloped aspect of the existing behavior of centers and their waves is the much more likely actual case.

Furthermore, it has long been hypothesized and has been verified by measurements to better than one part in one million accuracy that gravitational and inertial mass are identical in value. That would certainly seem to indicate that they are the result of very similar behaviors of the same wave propagation.

If gravitation is to be due to an aspect of the behavior of centers-of-oscillation and their propagated waves then it can be observed that the behavior of gravitation is slightly similar to the behavior of a negative electric field. The reasons for this observation are:

- the gravitational field must attract encountered masses, and
- since the principal mass in encountered masses resides in the atomic nuclei and the nuclei have a positive charge a negative field would appear to be required in order to have an attracting effect.

Gravitation might, therefore, be an aspect or effect due to negative electric field.

If it is assumed that the gravitational field is some form of quite weak negative electric field it remains to be seen how that slight negative electric field could produce gravitational attraction on all atoms that it encounters. After all, all atoms are overall charge-neutral. They should be expected to experience equal and opposite electrostatic attraction and repulsion by the effect of such a

negative electric field / gravitational field acting on their positive nuclei and negative orbital electrons.

Perhaps, when an atom experiences an external electric field the response of the overall atom is dominantly that of the nucleus. In a negative electric field an atom of matter would experience an attraction of the positive nucleus toward the source of the electric field. Perhaps, however, the orbital electrons do not experience an effective net repulsion. The reason for this phenomenon, if it operates at all, would be that the electric field effect is actually, of course, the repetitive arrival at and encountering of each orbital electron by the incoming U-waves.

The effect of arriving U-waves on orbital electrons could perhaps be to excite the electrons in the manner discussed in section 15 - *A Model for the Universe (5) - Quanta and the Atom*. An arriving U-wave pulse would have the expected effect on the orbital electron, delivering a change in energy and momentum to the electron. But, unless the arriving wave were large enough to cause the electron to change orbit or to become completely free of the atom, the net effect might only be that the electron re-radiates the wave and resumes its orbital path. Even if the arriving wave caused the electron to change its orbit or completely freed the electron from the atom the effect might still not be one of repelling the overall atom.

The gravitational field, if it were a very weak negative U-wave / electrostatic field, might agitate the atom's orbital electrons to no net effect and attract the positive atomic nucleus in the normal electrostatic fashion. Behavior in such a fashion, if it does occur, would:

- always be an attracting force because the gravitational field would be negative and the nuclei of the atoms encountered are positive;
- be according to the same *inverse square of the separation distance* law as is the case for the Coulomb interaction.

But, how could such a weak negative electric field come about ? One possible cause might be as follows.

The U-waves propagated outward by a center-of-oscillation do not all propagate outward forever. That portion of the wave front that encounters another center-of-oscillation under circumstances which result in acceleration of the encountered center is absorbed in the process of the electrostatic, the Coulomb, interaction, as already described in section 16 - *A Model for the Universe (6) - The Neutron, Newton's Laws*.

In an atom with its positive nucleus and negative orbital electrons the U-waves from each of those particles could be expected to be partially absorbed by some of the other particles in the atom. The net $+U$ field from the atom, observed from outside of the atom, could be expected to be somewhat less than that propagated by the positive nucleus. The net $-U$ field from the atom, observed from outside of the atom, could likewise be expected to be somewhat less than that as propagated by the negative orbital electrons.

The atom is overall electrically neutral in the sense of having equal total positive and negative charges among its particles. But, outside of the atom, the overall net U-wave field from the atom might not be electrically neutral as is

usually assumed. Perhaps it is slightly negative. That condition could occur if there were a difference in the amount of absorption of negative and of positive U-waves within the atom, if the positive field of the nucleus were to experience a greater reduction due to interception by the electrons of the atom than the reduction of the negative field of each of the electrons due to its partial interception by the other electrons and the nucleus. Since the nuclear cross-section is smaller than that of the electron such behavior might be expected.

Not all U-waves that encounter a center-of-oscillation are intercepted or absorbed, however. As presented in the discussion of Newton's laws earlier, if the interaction of incoming waves and the encountered center results in acceleration of the center, results in a change in the center's speed, the magnitude of its velocity, then the incoming waves are effectively absorbed. The "absorption" is actually the change in the propagation in that direction by the center, an action that the center is forced to take because of its change in velocity and consequent change in the propagation required of it to maintain wave field continuity.

The discussion of Newton's laws implicitly treated the case of change in speed, not change in direction. The general analysis is also valid with regard to a change in direction; however, it must be recognized that in a case where the incoming wave changes the direction but not the speed of the encountered center the directions or orientation of the encountered center's propagation change but not the magnitude or shape. The effect is only as if one were to reach in, grasp the center, and rotate it the appropriate amount.

In such a case, that is an acceleration of a center which produces change of direction but not change of speed, there is no net absorption of U-waves, only a change in the directions of propagation of the encountered center's various magnitudes in various directions, as if the center has been rotated from being oriented toward the initial direction to being oriented toward the final direction.

The orbital electrons of an atom are continuously accelerated. But that acceleration is only a change in the direction of their motion. It is the centripetal acceleration that maintains the electrons in their orbits. Without that acceleration each orbital electron would proceed on a straight line path tangential to its (old) orbit and away from the atom. The acceleration of each orbital electron is such as to continuously curve the electron's path into its correct orbit at the same, the orbital, speed. In elliptical orbits the speed does change, but the change is via a cyclic exchange of energy being stored in the electron's kinetic energy and its potential energy. Just as in a pendulum, there is no overall net speed change, only an oscillation.

Thus the atomic electrons do not absorb any of the incident U-waves coming from the nucleus as part of their normal orbital functioning within the atom. (Of course electron orbit changes or the electron's being freed from the atom do involve absorption of incoming waves, but those actions are the result of absorbing waves that come from outside of the atom involved, not its own nucleus.)

Unless the atom is perfectly symmetrical the nucleus, also rotates about the common center of mass of the nucleus and orbital electrons. But in that motion its behavior is analogous to that of the electrons and the acceleration does not involve the absorption of U-waves propagated from within the atom.

In short, the particles of an atom do not, to net effect, absorb any of the U-waves generated within the atom; the external electric field of the atom due to the charges within the atom is indeed neutral. The required (as tentatively hypothesized to account for gravitation) net negative electric field of the atom does not occur, is not caused by a difference in the absorption of the U-wave fields by the negative centers and the positive centers within the atom.

And, if it were the case that absorption of waves from within the atom resulted in a net negative field as tentatively hypothesized above, that field would be the difference of a negative orbital electron field, varying slightly in magnitude with the mass of the nucleus, and a positive nuclear field, varying slightly in magnitude with the number of orbital electrons. The net negative field variation with atomic mass would be curvilinear, not the straight line variation that the actual behavior of gravitation exhibits.

Furthermore, if the effect of a U-wave field, that is, an electrostatic field, on an atom were the normal Coulomb effect on the nucleus but only agitation of the orbital electrons to no net effect, then the consequences of that behavior should be observable under proper conditions. For example, one could use an instrument used in early investigations of static electric effects and called the electroscope. It consisted of two flat strips of metal foil (typically copper or silver) hanging, hinged independently but directly opposite each other, from a common narrow metal rod or shaft. Since the entire device is metal an electric charge placed anywhere on it tends to distribute over both of the two metal strips. (In practice the device is placed in a glass container to avoid air currents having an effect on results.)

The metal strips of the electroscope, having the same charge, repel each other in the usual Coulomb fashion. The result is that if the electroscope is uncharged the two metal strips hang straight down together in their normal relaxed position. But, if there is any charge on the device then the two metal strips tend to swing apart under the mutual repulsion experienced between the like charges. This occurs whether the charge placed on the electroscope is positive or negative. The device is, then, an indicator of the presence or absence of charge and can crudely indicate the relative amount of charge present by how far apart the strips tend to swing.

If such a charge, present on one of the metal strips of the electroscope, had a net effect (aside from that on the corresponding free charge on the other strip) only on the atomic nuclei in the other strip and not on those atoms' orbital electrons, then:

- a positive charge on the electroscope would cause each metal strip's charge to repel the positive nuclei in the other strip, an effect in agreement with observed results, but
- a negative charge on the electroscope would cause each metal strip's charge to attract the positive nuclei in the other strip, an effect opposite from the actual observed results.

Since the amount of charge in all of the nuclei of the atoms of one of the metal strips is much greater than the extra charge that can be placed on the electroscope in its operation, the effect of such placed extra charge in repelling its counterpart on the other metal strip would be much less than the effect of the added charge

on the atomic nuclei of the metal strip's atoms if it operated as hypothesized. But no such effect appears and the hypothesis is, therefore, invalid.

Then, why does the hypothesized difference in the behavior of nuclei versus orbital electrons in an external electric field not operate in fact? The reason is that individual cycles of U-waves, of electric field, are quite different from the photons that produce the usual orbital electron effects. The frequency of electron U-waves is on the order of 10^{20} Hz (cycles per second) and that of proton U-waves 10^{23} . The time for a single electron orbit is on the order of 10^{-16} to 10^{-15} seconds. Thus the electric field U-wave cycles encountering orbital electrons occur at the rate of on the order of 10^4 to 10^7 per orbit.

On the other hand, the photon oscillation is on the order of once per orbit. The U-wave field is almost continuous in its rapid impulsing of the encountered electron. The photon is one, sudden action, comparatively.

Furthermore, the U-wave is a simple medium oscillation of the $[1 - \cos]$ form. The photon is an electromagnetic wave, an imprint on the pattern of U-waves propagated by a charge, the imprint caused by the motion of the charge as presented in the discussion of electromagnetic radiation.

Clearly the entire foregoing hypothesis, a net external negative electric field from each atom and that field's having an effect on the atomic nuclei but without a corresponding net effect on the orbital electrons, is incorrect. Apparently the situation is indeed as would be classically expected: every atom and body of matter is normally charge neutral and, when charge neutral, exhibits a neutral external electric field. Every atom's components, the positive nucleus and the negative orbital electrons, respond to electric field in the expected equal but opposite manner.

(This rather tedious investigation of invalid effects has been undertaken because the questions treated can arise if not well laid to rest. The new structure of Universal Physics can raise new questions that need to be investigated and clarified as part of the development.)

The net external electric field of an atom is the sum of the individual negative field of each of the orbital electrons and the positive field of the nucleus. Taking, for the moment, the case of the most simple atom that has orbital electrons, ${}_1H^1$, there is the $+ [1 - \cos]$ form field of the nucleus (a single proton) at the nuclear (proton) frequency and the $- [1 - \cos]$ form field of the single orbital electron at the electron frequency. The waves that each propagate vary with time and with distance. That is, at any location in space the wave field varies with time according to its frequency and at any instant of time the wave field is a particular wave form in space with its characteristic wavelength.

The expressions for the two waves in terms of time, t , and distance, d , are as given in equations 19-2 and 19-3, below.

$$(19-2) \quad \text{Wave}_{\text{electron}} = -U_c \left[1 - \cos \left[2\pi f_e t - \frac{2\pi d}{\lambda_e} \right] \right] \cdot \frac{1}{d^2}$$

$$(19-3) \quad \text{Wave}_{\text{proton}} = +U_c \left[1 - \cos \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right] \cdot \frac{1}{d^2}$$

The proton frequency is much greater than that of the electron, 1836.152701 times greater at rest as presented earlier in this work. Likewise the proton wavelength is correspondingly much smaller than the electron wavelength, $1/1836.152701$ times smaller at rest.

The net wave propagation external to this atom is the sum of the above two waves. Since it is overall neutral on the average it has no net electrostatic effect. Since it is the only propagation from the atom it must be its gravitational field. That is as follows.

$$(19-4) \quad \text{Wave}_{\text{sum}}({}_1\text{H}^1) \equiv \text{Gravitational Field of } {}_1\text{H}^1 \\ = U_c \left[\text{Cos} \left[2\pi f_e t - \frac{2\pi d}{\lambda_e} \right] - \text{Cos} \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right] \cdot \frac{1}{d^2}$$

In equation 19-4 if the time, t , is set to some specific value then the two terms involving t become merely a phase angle and the expression is a space-varying difference of two cosines. In the equation if the distance, d , is set to some specific value then the two terms involving d become merely a phase angle and the expression is a time-varying difference of two cosines. In fact, if the $1/d^2$ is omitted in addition and d is set to zero, the expression is identical to that of equation 16-3 for the time-varying oscillation of the neutron, which is the sum of a proton and an electron as is equation 19-4.

(There is a small difference between the field of a ${}_1\text{H}^1$ atom and that of a neutron. In the case of the neutron the proton and electron are exactly co-located. In the Hydrogen atom the electron orbits the proton. At large distances from either source the slight difference between the waves should be essentially undetectable even though minutely present except that in addition the frequencies of the proton and the electron oscillations in the neutron are somewhat different from their values in the Hydrogen atom.)

In the case of the time-varying wave form at a specific location no inverse square effect along the wave form would be observed because d does not change. In the case of the space-varying wave form at a specific time there is an inverse square variation along the wave form; however, at significant distances from the source atom, the amount of such variation would be extremely gradual and essentially too small to observe. In either case the form of the wave form is such that its average value is zero so that no net Coulomb effect occurs due to the waves encountering a center.

There is, however, another as yet unconsidered real difference between the U-wave fields propagated from an atom by the negative electrons and the positive nucleus: their frequency content. Since an amplitude or sign difference appears to have been eliminated as a possible cause of gravitation, it would appear that the only other possible cause has to do with frequency.

The frequency content of the atom's negative and positive fields does correspond directly to the mass. For simple centers-of-oscillation the frequency is directly proportional to the center's mass. For the complex centers, all of the atomic nuclei except ${}_1\text{H}^1$, the nuclear wave form is a complex of frequencies and the overall frequency content is progressively higher as the mass increases (per equation 17-2) the wave form's shape corresponding directly to mass. If there were an effect in the interaction of a wave with a center which effect were directly proportional to the frequency and produced a force in the attracting direction, that could be gravity.

All of the treatment of the focusing of incoming waves by an encountered center, an effect due to the gradient of medium in the space that the incoming wave must traverse as it approaches the encountered center, has deemed that medium gradient to be due solely to the propagation of the encountered center. But, the encountered center's propagation is not the only cause of medium in the region. The incoming wave also contributes medium.

One might then argue as follows. An infinitesimal "piece" of incoming wave "looking around to observe the gradient ahead, behind, to the right, to the left" so as to "decide how much its course is to be there deflected" does not "know" whether that gradient ahead, behind, etc. is due to the encountered center or is another so infinitesimal "piece" of the incoming wave, the "piece" just before or just after or just next to "itself". The amount of medium at a location in space at an instant of time is the amount there, regardless of how it came to be there. The gradient depends on the variation in such amounts in space.

The incoming wave could change the gradient from that of the encountered center's wave field, only, to a slightly differing gradient. When that effect is omitted, as until now, the usual Coulomb's law effect results. Therefore, the inclusion of that effect should appear, relatively, as a new effect. The direction of this effect's action is such as to reduce the Coulomb effect when the Coulomb effect is repulsion and to enhance the Coulomb effect when the Coulomb effect is attraction. In other words this marginal additional effect seems to modify the normal Coulomb effect in a manner that appears always as net attraction. This is demonstrated in equations 19-5, 6, 7, and 8, below.

The analysis will be done using the interaction between two protons separated by the distance D . Because the focusing action takes place significantly only in the half wavelength adjacent to the encountered center, the inverse square variation along the incoming wave within that region can be neglected, it being a quite minute variation at the distance between the two centers.

Therefore, the incoming wave magnitude at the encountered center is taken as constant at the inverse square of D . As presented in section 16 - *A Model for the Universe (6) - The Neutron, Newton's Laws*, the inverse square effect on the encountered center's own propagated waves cancels with other effects for focusing purposes and can be neglected.

On that basis the equations of the waves are as follows (the interaction of two protons separated by distance D).

$$\begin{aligned}
 (19-5) \quad U_e &= U_c \cdot \left[1 - \cos \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right] && \text{Encountered} \\
 &&& \text{center's} \\
 &&& \text{propagation} \\
 U_i &= \frac{U_c}{D^2} \cdot \left[1 - \cos \left[2\pi f_p t + \frac{2\pi d}{\lambda_p} \right] \right] && \text{Incoming} \\
 &&& \text{wave} \\
 &= U_w \cdot \left[1 - \cos \left[2\pi f_p t + \frac{2\pi d}{\lambda_p} \right] \right]
 \end{aligned}$$

The signs of the terms in "d" are opposite because the incoming wave is traveling in the opposite direction to the propagation from the encountered center.

It is convenient here to break the encountered center's propagation into two parts: one part equal in magnitude to the incoming wave, the other part the balance of the propagation.

$$(19-6) \quad U_e = U_w \cdot \left[1 - \text{Cos} \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right] + \dots \\ \dots + U_c \cdot \left[1 - \frac{U_w}{U_c} \right] \cdot \left[1 - \text{Cos} \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right]$$

The sum of the incoming wave of equation 19-5 and the encountered center per equation 19-6, U_Σ , is

$$(19-7) \quad U_\Sigma = U_w \cdot \left[1 - \text{Cos} \left[2\pi f_p t + \frac{2\pi d}{\lambda_p} \right] \right] + \dots \\ \dots + U_w \cdot \left[1 - \text{Cos} \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right] + \dots \\ \dots + U_c \cdot \left[1 - \frac{U_w}{U_c} \right] \cdot \left[1 - \text{Cos} \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right]$$

The gradient in space as used for the Coulomb effect is the rate of change with respect to d of U_e of equation 19-5, the encountered center alone. It is given in equation 19-8a, below.

The total gradient, taking account of the incoming wave's contribution to medium is the rate of change of the sum expression, equation 19-7, with respect to d . The rate of change is, of course, the first derivative (detail notes *DN 1 - Differential Calculus, Derivatives*). Its value for the first line of equation 19-7 is identical to that for the second line except that it is of opposite sign, so the first two lines cancel. The remaining expression for the gradient is, then as given in equation 19-8b.

$$(19-8a) \quad \text{Gradient}_{\text{Coulomb}} = U_c \cdot \left[1 \right] \cdot \left[\frac{2\pi}{\lambda_p} \cdot \text{Sin} \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right]$$

$$(19-8b) \quad \text{Gradient}_{\text{Total}} = U_c \cdot \left[1 - \frac{U_w}{U_c} \right] \cdot \left[\frac{2\pi}{\lambda_p} \cdot \text{Sin} \left[2\pi f_p t - \frac{2\pi d}{\lambda_p} \right] \right]$$

The gradients differ by the factor $\Delta = U_w/U_c = 1/D^2$, the total gradient (Coulomb effect plus the tentatively hypothesized gravitational effect) being less for this case of two protons. Less gradient means less incoming wave focused onto the encountered center's singularity. That means less acceleration produced, which means less force applied since the encountered center, itself and its mass, have not changed. Thus the total repulsive force between the two protons is less because of the U_w/U_c gradient reduction which produces that much gravitational attraction in relative effect.

If now the sign of either the encountered center or the incoming wave of equation 19-5 is changed to minus, so that the interaction is of a $+U$ center with a $-U$ center, then all of the above development is the same except that the $[1 - U_w/U_c]$ factor becomes $[1 + U_w/U_c]$, so that the total gradient is greater, enhanced rather than reduced as it was for the centers both being $+U$.

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Since the interaction of a $+U$ center with a $-U$ center is Coulomb attractive, the enhancing of the force is that much (the enhancement amount) gravitational attraction in relative effect. (The case of two $-U$ centers is similar to that of two $+U$ centers.)

If a ${}^1_1\text{H}^1$ atom were to act on a proton in this manner the effect would be as in Table 19-1 below. The Coulomb repulsions and attractions would cancel; the attracting gravitational modifications would add.

	Relative ${}^1_1\text{H}^1$ Effect on a Proton:	
	<u>Repulsion</u>	<u>Attraction</u>
(1) The Hydrogen atom's proton repels the encountered proton with a normal Coulomb force.	1	
(2) The gravitational effect per equation 19-8 reduces that Coulomb repulsion.		Δ
(3) The Hydrogen atom electron attracts the encountered proton with a normal Coulomb force (the electron charge is the same as that of the proton so the force magnitude is the same).		1
(4) The gravitational effect per equations 19-8 enhances that Coulomb attraction.		Δ
(5) The net force.		$2 \cdot \Delta$

Table 19-1

However, the incoming wave cannot interact with itself as here hypothesized since all parts of it are moving at the same speed. The focusing effect of gradient depends on the U-waves that are being focused passing through the other U-waves whose gradient in space causes the focusing. (The bending of light by a gravitational field occurs when the light passes by a massive gravitating body, which passing is traveling essentially perpendicular to the body's radial field or perpendicular to a component of that field.)

While, if the foregoing effect of direct modification of the focusing gradient of the encountered center by the incoming waves were operative, it would seem to produce an effect like gravitation, that effect in fact does not occur at all. The focusing U-wave field gradient that the incoming wave travels through is that of the encountered center unmodified by the incoming wave (so far as this effect is concerned) because the incoming wave cannot travel through itself.

Furthermore, since the (invalid) hypothesized effect is proportional to $\Delta = U_w/U_c = 1/D^2$ and the effect is a factor slightly changing the ordinary Coulomb effect, which itself is proportional to $1/D^2$, then, if the effect were not invalid, it would therefore be proportional to $[1/D^2]^2 = 1/D^4$, which, of course, is not the behavior of gravitation.

Two seemingly potential causes of gravitation have now been hypothesized and each has been found to be not operative in reality (and, also,

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Newton's law of gravitation, equation 19-1) results in a peculiar effect: all bodies at the same location in a gravitational field (at the same distance from the source of the gravitational field and therefore experiencing the same magnitude of that field) experience the same acceleration. Usually, if a force is applied to a collection of bodies of varied masses one finds that each body accelerates in inverse proportion to its mass. The more massive body has small acceleration as a result of a given force; a less massive body is accelerated more by the same force. That is the effect of the action of Newton's second law of motion, $F = M \cdot A$.

But in the case of gravitation the " F " in $F = M \cdot A$, itself, is also proportional to the mass, " M ". As a result all bodies are accelerated equally in a gravitational field regardless of the bodies' masses. This does not mean that gravitational acceleration is constant. It is constant at any given distance from a particular gravitationally attracting mass, but the acceleration depends on that distance, per the D^2 in the denominator of equation 19-1, and is different for different values of D .

If the acceleration of all masses is the same (when being acted upon by the same magnitude of gravitational field) then, in the present hypothesis for gravitation, each of the centers-of-oscillation in those masses experiences the same ratio of the amount of incoming wave actually focused onto its singularity to the center's oscillation amplitude. This conclusion results directly from the effect set out in equations 16-11 through 16-14. Acceleration is the velocity change that is made imperative by a change in the amount of center propagation necessary to maintain continuity of that propagation, the change being due to the incoming wave's proportion to the center's otherwise propagation.

The gravitational focusing hypothesis is that the incoming wave minutely changes the concentration of the encountered center's propagation in the region where the incoming wave is focused by that propagation. It would do so by the action of its (there) very weak focusing field at the encountered center, which is at distance D from its source center, where its focusing field is the usual focusing field of a center as developed in section 16 - *A Model for the Universe (6) - The Neutron, Newton's Laws*. Whatever shape, magnitude and consequent focusing effect the encountered center's propagated wave field would otherwise have applied to the incoming wave is either enhanced -- multiplied by a factor slightly greater than one, or reduced -- multiplied by a factor slightly less than one. That change is always proportional to the encountered center's natural (before or without any gravitational action) wave propagation because it is a focusing action acting on that natural propagation.

It is for that reason that this effect would produce the same acceleration in all centers that it encounters at the same distance, D . The encountered center's own focusing would be enhanced or reduced by the focusing effect of the arriving incoming wave. That incoming wave focusing effect is the same amount of focusing (at any particular D), the same amount of bending of each path of waves passing through it, regardless of the size or type of the center whose waves it acts on, regardless of the amount of those waves. Applied to, acting on, the encountered center it would produce a directly proportional change in the focusing power of that encountered center and, consequently, in the amount of incoming wave that that encountered center would focus onto itself.

The mass of a center-of-oscillation in this Universal Physics' terms is the amplitude of the center's oscillation relative to its average focusing. The

amplitude is that of the oscillatory part of the wave form and the average focusing is the result of the shape of the wave form. Therefore, the propagated waves from a center carry that mass datum with them and, before any inverse square decrease, they represent / are that mass. In the pure Coulomb effect only the amplitude of those waves is employed in determining the Coulomb force that results. In the hypothesized gravitational action, however both the amplitude and the focusing would enter into modifying the encountered center's own focusing. The focusing of the encountered center would be modified in proportion to the source center's mass.

The source center's waves, decreased according to the inverse square effect, correspond to m_s/D^2 at the encountered center (m_s is the mass of the source center). By this hypothesized gravitational focusing effect they produce the same amount of acceleration in all centers that they encounter at the same distance, D . Denote, for the moment, the amount of that effect by g ; that is,

$$(19-9) \quad a_g = g \cdot \frac{m_s}{D^2}$$

$a_g \equiv$ gravitational acceleration magnitude
 $a_c \equiv$ Coulomb acceleration magnitude
 $a_t \equiv$ total acceleration magnitude
 $a_t = a_c \pm a_g$ ("+" when Coulomb is attractive
 "-" when Coulomb is repulsive)

Per Newton's laws of motion,

$$(19-10) \quad a_g = \frac{F_g}{m_e} \equiv \frac{\text{Gravitational Force}}{\text{Mass of Encountered (accelerated) Center}}$$

so that, by combining equations 19-9 and 19-10,

$$(19-11) \quad \frac{F_g}{m_e} = g \cdot \frac{m_s}{D^2} \quad \text{or, rearranging} \quad F_g = g \cdot \frac{m_s \cdot m_e}{D^2}$$

which is of the same form as Newton's law of gravitation.

However, there are two defects in this hypothesis. The first is that it can only produce either enhancement of all interactions (net focusing) or degradation of them all (net defocusing), that is it could either increase both all repulsions and all attractions or decrease both. Gravitation requires an effect that modifies the Coulomb effect by decreasing all repulsions and increasing all attractions to yield an overall net attraction.

Secondly, the magnitude of the incoming wave at the encountered center is proportional to $1/D^2$, and the hypothesized gravitational focusing effect is a factor that is proportional to that same incoming wave magnitude. Therefore, if the effect were valid, it would produce gravitation that is proportional to $[1/D^2]^2 = 1/D^4$, which, of course, is not the behavior of gravitation. (This same defect was encountered with the prior hypothesis.)

This effect, gravitational focusing, therefore joins its predecessor hypothesized causes of gravitation as invalid. But in this case, the invalidity is only due to the effect's not conforming to all of the behavior of gravitation. The effect nevertheless does occur. The only reasonable explanation of that conflict

or contradiction is that the effect must be so minute, even compared to gravitation, which is quite minute itself, that the effect is essentially undetectable.

That conclusion is not unreasonable. Since the effect is not always that of attraction, but, rather, a mere quite minor change in the Coulomb effect, it tends to average out overall and locally (down to the atomic level) just as the Coulomb forces do. Furthermore, Coulomb focusing actually is a quite minute effect. The "focusing cone" (Figure 16-7) is very narrow, which means that the focusing gradients are quite weak, and that weakness is immediately adjacent to the center providing the gradient, normally the encountered center. At distance D away from that center, as for the now invalid hypothesis of gravitation, the focusing gradients are immensely weaker.

In addition, the gravitational focusing would be an alternation of focusing and defocusing. If δ is the minute fractional change that the gravitational focusing produces in the normal Coulomb focusing, then it produces a cyclical alternating shift in the Coulomb gradient between $[1 + \delta]$ and $[1 - \delta]$ times it. Since focusing depends upon the square of the gradient the amount of the modification of the Coulomb focusing by the gravitational focusing would be as follows on the average.

$$\begin{aligned} & \left[[1 + \delta]^2 + [1 - \delta]^2 \right] \div 2 \\ &= \left[[1 + 2\delta + \delta^2] + [1 - 2\delta + \delta^2] \right] \div 2 \\ &= 1 + \delta^2 \end{aligned}$$

where, since δ before squaring is minute, squared it would, likely, be so extremely minute as to be not detectable.

However, this latest unsuccessful attempt to find the cause of gravitation does perform the useful function of emphasizing the essential characteristics of the effect to be sought, of whatever effect really is the cause of gravity.

The effect must either:

- change the amount of (Coulomb) incoming wave focused onto the encountered singularity, or
- change the amount of the (Coulomb) reaction of the center to the same amount of incoming wave so focused, or
- be the result of a new type of reaction of the center to the incoming wave;

and it must do so (whichever it does) in a manner that:

- produces the same acceleration on all masses encountered at the same distance, D , (the acceleration must be independent of the mass of the encountered center),
- avoids the problem encountered more than once in the preceding, the $1/D^4$ problem,
- is directly proportional to the incoming wave's frequency or, which is the same thing, inversely proportional to its

The slowing is the same slowing that produces deflection of a ray of propagating wave if the wave passes through a U-wave gradient across its path, the usual focusing action. But, even a ray of propagating wave that is exactly on the line joining the two centers and that consequently experiences no deflecting gradient across its path is nevertheless slowed by its passage through other waves. It is not deflected because the slowing on all sides of it is the same, but it is slowed.

This effect is quite small (as is gravitation) because usually the slowing of propagated U-waves when passing through other such waves is quite small. But the effective result is that the encountered center, forced to propagate at a slightly lesser wave velocity toward the source center, must then move in that direction, its propagation in the opposite direction being forced to accommodate accordingly.

That is, a center always propagates forward and rearward as is required by its then state of motion, as described in section 16 - *A Model for the Universe (6) - The Neutron, Newton's Laws*. If an external action, incoming waves, forces a change in that propagation then the state of motion changes, must change, along with the forward and rearward propagation so that all are again consistent with each other.

If the encountered center's wave velocity toward the source center is slowed in the amount v then the center must:

- (1) Propagate toward the source (to be the forward direction) at

$$c' = c - v$$

which requires that

- (2) the center, to maintain continuity of medium forward, must move toward the source (forward) at velocity

$$v$$

which further requires that

- (3) the center, to maintain continuity of medium rearward, must propagate away from the source (rearward) at

$$c'' = c + v$$

The effect in terms of the descriptions for Newton's laws of motion of Figures 16-6(a) through (c) is as in Figure 19-4, on the following page.

The reasoning of equations 16-10 through 16-14 applies to this phenomenon with the quantity U_w of those equations replaced with U_g , the amount that the encountered center's normal propagation must change because of the effect of this new circumstance.

The overall effect in changing the center's propagation is as follows:

(19-13) Forward

$$\begin{aligned} \Delta U_{e'} &= \Delta U_e - \Delta U_g \\ &= \Delta U_e \cdot \left[1 - \frac{\Delta U_g}{\Delta U_e} \right] \end{aligned}$$

Rearward

$$\begin{aligned} \Delta U_{e''} &= \Delta U_e + \Delta U_g \\ &= \Delta U_e \cdot \left[1 + \frac{\Delta U_g}{\Delta U_e} \right] \end{aligned}$$

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The consequent momentary acceleration in each of those instances would be different, but over a large number of cases the net average acceleration would be independent of that factor, independent of the encountered center's frequency and wavelength of oscillation and of how they relate to the source center's period.

The result is an acceleration of

$$\begin{aligned}
 (19-18) \quad a_{grav} &= \frac{\Delta v}{T_w} = \frac{\Delta v}{T_s} && [T_w \text{ is the period of the incoming waves.} \\
 & && T_s \text{ is the (identical) period of the source center, itself.}] \\
 &= \Delta v \cdot f_s && [f = 1/T]
 \end{aligned}$$

Being directly proportional to the source center's frequency, f_s , makes a_{grav} be directly proportional to the source center's mass as the gravitational acceleration must be. Because the slowing, Δv , is directly proportional to the amplitude at the encountered center of the source center's waves, a_{grav} exhibits the inverse square reduction as expected and required.

However, it could be argued that the encountered center's wave field focuses the incoming wave onto the encountered center and that that should make the effect dependent on the encountered center's mass as in the Coulomb effect. But, focusing is of the medium flow. The amount of that focusing action depends on local wave variations in the encountered medium amount, the head or potential. Medium amount is not focused; it causes focusing. And through the same mechanism as that by which the encountered medium amount produces focusing the source medium amount produces gravitational slowing of the encountered propagation. That is the cause of gravitation.

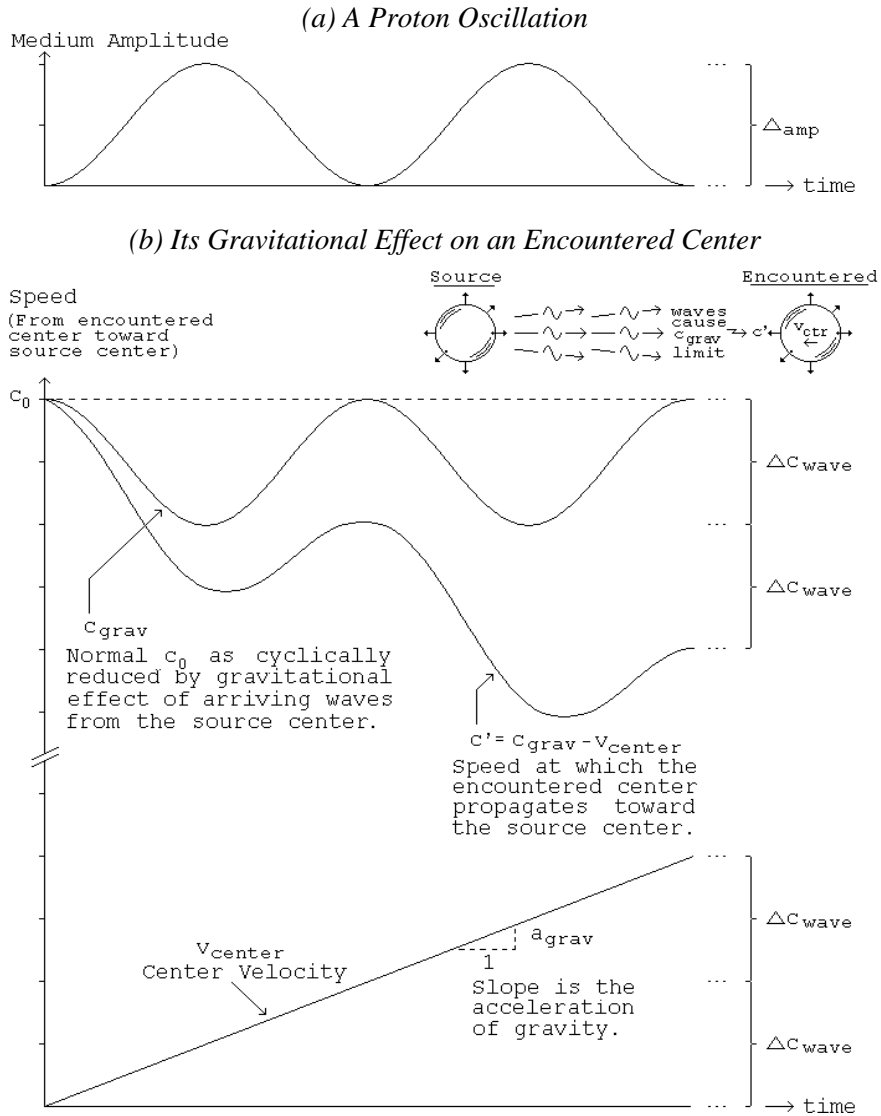
The mechanism of slowing when U-waves pass through other U-waves was presented in the earlier equations 16-27 through 16-35. While Coulomb effects depend on the $+U/-U$ polarities involved, U-wave slowing and focusing are independent of that polarity. That is because the slowing depends on the effect that μ and ϵ have on the speed of propagation and μ and ϵ are pure positive scalar quantities. As a result, this gravitation effect always produces attraction between the source and encountered centers by the slowing of the propagation of the encountered center's waves toward the source center.

This effect, then, exhibits all of the requisite behavior to qualify as gravitational acceleration. Furthermore, the effect is of the same kind as the bending of light rays by large cosmic masses, in that both are due to U-waves being slowed by passing through other U-waves. Since both are gravitational effects their mechanisms should be the same.

Recalling the sinusoidal zero-to-maximum-to-zero form of the waves, then in gravitational action a cycle of wave arriving from a distant source center produces a temporary pulse of local increase in the values of μ and ϵ at the location on the encountered center that is toward the source center. That reduces the speed at which the encountered center can propagate in the direction of the source center. That imbalance forces the encountered center to make the necessary changes in its propagation and velocity. This is depicted graphically in Figure 19-5 on the following page.

Essentially, the encountered center "might like to" only change its speed of propagation toward the source center. But, it cannot do that without taking on

additional velocity toward the source center. Its velocity change then further modifies the corrective change required in the speed of the center's propagation in that direction. Those two tendencies arrive at a mutual accommodation; however, the entire event is in continuous flux as the arriving wave form proceeds through its various values.



*Figure 19-5
Gravitation*

The discussion has been in terms of the waves from the source center encountering and producing their effects on the encountered center. Of course, as has been emphasized before, all of the effects are mutual. Each of the centers performs in both roles, as source and as encountered center, all of the time and simultaneously. At the same time that a center's waves are being required to propagate at a reduced speed by the effect on them of incoming waves,

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gravitational action on the center that also results in center acceleration, those waves of the center are acting similarly on the other centers and producing similar effects upon arriving there.

ANALYTICAL DERIVATION OF NEWTON'S LAW OF GRAVITATION

Examining the laws governing gravitation and the Coulomb effect and comparing them,

(19-19)
$$F_g = G \cdot \frac{m_1 \cdot m_2}{d^2}$$
 Newton's law of gravitation in SI units

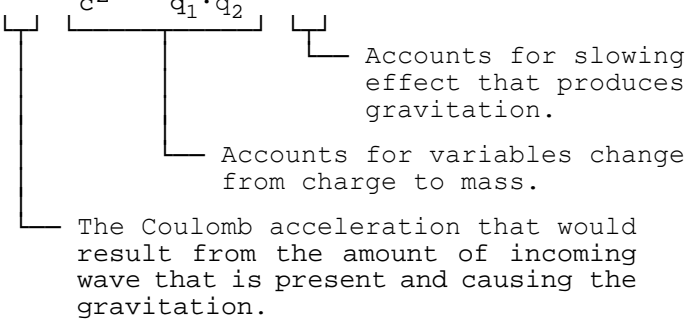
(19-20)
$$F_c = c^2 \cdot \frac{q_1 \cdot q_2}{10^7 \cdot d^2}$$
 Coulomb's law in the same SI units

then

(19-21)
$$F_g = F_c \cdot \frac{10^7}{c^2} \cdot \frac{m_1 \cdot m_2}{q_1 \cdot q_2} \cdot G$$

and, since $F_g = m_2 \cdot a_g$ and $F_c = m_2 \cdot a_c$, then

(19-22)
$$a_g = a_c \cdot \frac{10^7}{c^2} \cdot \frac{m_1 \cdot m_2}{q_1 \cdot q_2} \cdot G$$



Accounts for slowing effect that produces gravitation.

Accounts for variables change from charge to mass.

The Coulomb acceleration that would result from the amount of incoming wave that is present and causing the gravitation.

Equation 19-22 indicates that for the dimensional units of quantities to be correct and consistent then the units of G must be the units of

$$[c^2 \cdot q^2 / 10^7 \cdot m^2]$$

because only then will the units of a_c and of a_g in equation 19-22 be identical as they must. That is the case as follows.

(19-23) $G = 6.67259 \cdot 10^{-11} \text{ meter}^3 \text{ kilogram}^{-1} \text{ sec}^{-2}$

(per the previously referenced CODATA Bulletin)

Units of $[G] = L^3 \cdot M^{-1} \cdot T^{-2}$ where: $L \equiv \text{length}$
 $M \equiv \text{mass}$
 $T \equiv \text{time}$

$$\text{Units of } \left[\frac{c^2 \cdot q^2}{m^2} \right] = \frac{[L \cdot T^{-1}]^2 \cdot [M \cdot L]}{M^2}$$

$$= L^3 \cdot M^{-1} \cdot T^{-2}, \text{ the Units of } [G].$$

In other words, in terms of units, the G in equation 19-19, Newton's law of gravitation, is there to perform the function of converting from expressing in the terms of mass (gravitation, m^2) to in the terms of charge (Coulomb, $c^2 \cdot q^2$). That is, the behavior of gravitation in its effect on the encountered center is similar to the behavior of the Coulomb effect (incoming wave forcing a change in the encountered center's propagation of its U-waves), modified to be in terms of mass variables instead of charge variables and inherently always attracting.

However, the numerical value of G is

$$G = 6.67259 \cdot 10^{-11}$$

a constant, whereas the numerical value of

$$[c^2 \cdot q^2 / 10^7 \cdot m^2]$$

depends on what value of m is used. Clearly, there is more to the numerical value of G than just that expression. Just as clearly, whatever other quantity enters into that numerical value must be dimensionless since the units of the expression

$$[c^2 \cdot q^2 / 10^7 \cdot m^2]$$

are the same as the units of G . Furthermore, that other quantity, in combination with that expression for G must yield an overall quantity, a value for G , which is truly a constant and is independent of mass or other variables.

Taking the case of a proton, for which the m is m_p , we then have that G must be given by the following (still in SI units).

$$(19-24) \quad G = \frac{c^2 \cdot q^2}{10^7 \cdot m_p^2} \cdot [?] \\ = \frac{c^2 \cdot q^2}{10^7 \cdot m_p^2} \cdot \left[\frac{m_p^2}{m_*^2} \right]$$

[The m_p^2 is to cancel the other m_p^2 so as to make a true constant that is not dependent on a mass that is variable.]

[The ratio m_p^2/m_*^2 is to make the expression [?] stay dimensionless.]

[The m_* is a new constant -- a reference mass that is developed below.]

$$G = 2.307,079,555,52 \cdot 10^{-28} \cdot \left[\frac{1}{m_*^2} \right] \quad \text{[Taking the "c" and "q" values from CODATA]}$$

$$m_*^2 = 3.45755 \cdot 10^{-18} \quad \text{[Solving for } m_* \text{ and taking the "G" value from CODATA.]}$$

$$m_* = 1.85945 \cdot 10^{-9} \text{ kg (kilograms)}$$

Mass can be expressed in terms of the center-of-oscillation frequency, of course, and a value of an f_* can be obtained using the above method. In a

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similar manner, since a wavelength can be obtained from a frequency by using $c = f \cdot \lambda$, a value for λ_* can be obtained. Finally, the time period of the oscillation is the reciprocal of the frequency so that a T_* is obtained. These values are as follows.

$$(19-25) \quad \begin{aligned} f_* &= 2.52214 \cdot 10^{41} \text{ Hz (cycles per second)} \\ \lambda_* &= 1.18864 \cdot 10^{-33} \text{ m (meters)} \\ T_* &= 3.88189 \cdot 10^{-42} \text{ s (seconds)} \end{aligned}$$

The quantities m_* , λ_* , and T_* are close in value to the three defined "Planck constants" (as distinct from the Planck constant, h), the "Planck mass", "Planck length", and "Planck time": m_{Pl} , l_{Pl} , and t_{Pl} , which are as follows (per the CODATA bulletin).

$$(19-26) \quad \begin{aligned} m_{Pl} &\equiv \left[\frac{h \cdot c}{2\pi \cdot G} \right]^{1/2} &&= 2.17671 \cdot 10^{-8} \text{ kg} \\ l_{Pl} &\equiv \frac{h}{m_{Pl} \cdot c} \equiv \left[\frac{h \cdot G}{2\pi \cdot c^3} \right]^{1/2} &&= 1.61605 \cdot 10^{-35} \text{ m} \\ t_{Pl} &\equiv \frac{l_{Pl}}{c} \equiv \left[\frac{h \cdot G}{2\pi \cdot c^5} \right]^{1/2} &&= 5.39056 \cdot 10^{-44} \text{ s} \end{aligned}$$

(These Planck constants are not so much useful constants found to occur in known natural processes as quantities of expected or anticipated significance. In the view of quantum mechanics, when one gets down to the most minute quantities conceivable one is at the fundamentals of quantization. The constants were invented with that point of view in mind it being hoped, for example, that when separation distances between masses get down to on the order of the Planck length gravity will exhibit quantum behavior. No such success has yet developed in 20th Century physics because of its lack of a solution to the problem of gravity.)

One of those constants, the Planck length, l_{Pl} , (slightly modified because of an error in the original defining of the Planck constants) turns out to be quite important and fundamental as will be developed shortly below. Comparing the length / wavelength parameters from the above the following is obtained.

$$(19-27) \quad \begin{aligned} l_{Pl} &= \left[\frac{h \cdot G}{2\pi \cdot c^3} \right]^{1/2} && \text{[From 19-26, above]} \\ \lambda_* &= \frac{c}{f_*} = \frac{c}{m_* \cdot c^2 / h} && \text{[} h \cdot f = m \cdot c^2 \text{]} \\ &= \left[\frac{10^7 \cdot h^2 \cdot G}{c^4 \cdot q^2} \right]^{1/2} && \text{[From 19-24, above]} \\ &= \left[\frac{h \cdot G}{2\pi \cdot c^3} \cdot \frac{10^7 \cdot h}{2\pi \cdot c \cdot q^2} \cdot 4\pi^2 \right]^{1/2} && \text{[rearranging]} \\ \lambda_* &= l_{Pl} \cdot \frac{2\pi}{\alpha^{1/2}} && \text{[}\alpha \text{ is the "fine structure constant" treated in the previous Section 15]} \end{aligned}$$

From this G can then be restated from its form in equation 19-24 (or by solving equation 19-26 for G) as

$$(19-28) \quad G = \frac{c^4 \cdot q^2 \cdot 4\pi^2}{10^7 \cdot h^2 \cdot \alpha} \cdot l_{p1}^2 = \frac{2\pi \cdot c^3}{h} \cdot l_{p1}^2$$

Of course, the "Planck constant", l_{p1} , still remains to be defined and determined; that is, defined and determined on its own basis, not as set out above in equation 19-26 in terms of G . However, it would appear that it is a more "fundamental" constant than G , which would appear to be a combination of various fundamental constants per equation 19-28 (just as, for example, α is a combination of the fundamental constants μ_0 , h , c , q , and π).

Equations 19-17 and 19-18 joined are restated below as equation 19-29.

$$(19-29) \quad a_{\text{grav}} = \Delta v \cdot f_s = \Delta c \cdot f_s$$

Newton's law of gravitation expressed in terms of m_s and m_e rather than m_1 and m_2 , and with both sides of the equation divided by m_e , is

$$(19-30) \quad a_{\text{grav}} = G \cdot \frac{m_s}{d^2}$$

It can then be reasoned as follows.

$$(19-31) \quad \Delta c \cdot f_s = G \cdot \frac{m_s}{d^2} \quad [\text{Equating 19-29 and 19-30}]$$

$$\frac{m_s}{m_p} \cdot f_p \cdot \Delta c = G \cdot \frac{m_s}{d^2} \quad [\text{Frequency is proportional to mass and } f_p \text{ and } m_p \text{ are the proton frequency and mass, that is } f_s = f_p \cdot (m_s/m_p).]$$

$$f_p \cdot \Delta c = \frac{G \cdot m_p}{d^2} \quad [\text{Rearrangement}]$$

Then:

$$(19-32) \quad f_p \cdot \Delta c = \frac{G}{d^2} \cdot \frac{h \cdot f_p}{c^2} \quad [m = h \cdot f / c^2]$$

$$\Delta c = \frac{G}{d^2} \cdot \frac{h}{c^2} \quad [\text{Solve for } \Delta c]$$

whence:

$$(19-33) \quad \Delta c = \left[\frac{2\pi \cdot c^3}{h} \cdot l_{p1}^2 \right] \cdot \frac{1}{d^2} \cdot \frac{h}{c^2} \quad [\text{Substituting equation 19-28 for "G" in the above.}]$$

$$= \frac{2\pi \cdot c}{d^2} \cdot l_{p1}^2 \quad [\text{Simplifying}]$$

and

$$(19-34) \quad \frac{\Delta c}{c} = 2\pi \cdot \frac{l_{p1}^2}{d^2} \quad [\text{Rearranging}].$$