SECTION 13

A Model for the Universe (3)
Motion and Relativity

The motion of a center-of-oscillation involves an apparent contradiction, one to which there is a simple solution, and the solution to which provides a new insight into the phenomena treated in 20th Century physics under the heading Theory of Relativity. In order to approach the problem, it will be addressed initially in the terms of traditional 20th Century physics.

As with the Theory of Relativity, the problem of motion will be treated first at constant velocity, the case called Special Relativity. However, even for motion involving acceleration, at any moment of time the velocity is a constant. In other words, the constant velocity case applies to all cases of motion although for non-constant velocity cases additional considerations are involved.

The special aspect of constant velocity is that at constant velocity one cannot detect absolute motion, that is, one can say that there is a relative difference of velocity between two systems in one of which the observer is located, but the observer cannot say which system is moving and which, if any, is at rest. (Most persons have had the experience of being in one of two trains or vehicles that are so close to each other that looking out the window to the side from one train to the other only the other train is seen, and of then experiencing at least momentary doubt as to whether the one seen or the one ridden in is moving. The doubt is momentary because the observer here has other information and because usually the velocity is not perfectly constant.)

If one is in an accelerated system, that fact is apparent since acceleration produces detectable effects (braking for example). Velocity appears to the observer experiencing it to be relative but acceleration is absolute. Thus for the case of constant velocity it would appear that there is no prime overall reference to which all motion can be related.

The contradiction is as follows.

On the one hand

> (1) Regardless of the apparent absence of a functional prime reference relative to which all motion could be related, nevertheless the motion of a center-of-oscillation, motion while it is propagating waves in all directions, must inevitably affect the pattern of propagated waves. (This situation is related to the Doppler effect, the effect, for example, on a vehicle horn or siren as it passes by, in which the tone received by the observer first rises and then falls as the vehicle approaches and departs.)

But on the other hand

> (2) It is an indisputable verified fact that the velocity of light (and, therefore, the velocity of U-wave propagation) appears constant at the value c to all observers regardless of their frame of reference or state of motion.
>
> (3) It is also a so indisputable (and logically and reasonably necessary) fact that the laws of physics, the physical behavior of all reality, are identical as observed in all frames of reference. To an observer in a system at constant velocity, constant velocity motion of the observer's own or any other observed system cannot change the behavior of physical reality in the system. Otherwise we would have a chaos of different physics in different situations rather than our reliable single set of physical laws for the entire universe.
>
> (4) Since a center-of-oscillation is in effect a "system" or a "frame of reference" the foregoing must apply to all individual centers-of-oscillation.

Starting first without the above constraints, the initial expectation for the effect of center-of-oscillation motion on its propagated waves would be that the wave propagated in the direction of the center's motion would travel at the sum of the velocity of the center and the velocity of waves as propagated by a stationary center, and the wave propagated to the rear would be at the difference of the velocities. See Figure 13-1, below, which depicts an "idealized" center-of-oscillation moving at velocity v.

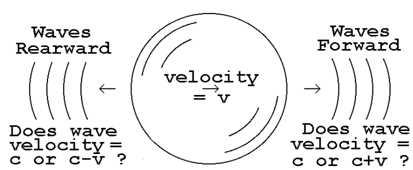

Figure 13-1

But, per (2), above, the wave velocity must always be c, the speed of light.

The next alternative would be the Doppler effect (that which occurs with the above horn or siren example), that the wave in the forward direction travels at velocity c but with increased frequency and reduced wavelength and the wave in the rearward direction also travels at velocity c but with reduced frequency and increased wavelength. But this also violates the constraints of the theory of relativity, making the velocity absolute not relative.

The solution of traditional 20th Century physics to the above dilemma is the Lorentz Transforms. In a mathematical sense these are equations, mathematical relationships, for transforming physical reality descriptions from one constant velocity system to another system at a different constant velocity. In a practical sense they are recognition of the fact that, if the observed velocity of light, c, is to be constant regardless of the observer's velocity, v, then the standard of measurement of time and length (the elements of velocity) must be different for different values of v. The mathematical statements for this, called the Lorentz Contractions, and their physical significances are as follows.

(13-1) $$L = L_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$$ [Observed length, in the direction of motion, shortens.]

(13-2) $$f = f_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$$ [Observed frequency slows.]

(13-3) $$t = t_r \cdot \frac{1}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$$ [Observed time periods lengthen, time passes more slowly.]

(13-4) $$m = m_r \cdot \frac{1}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$$ [Observed mass increases.]

[In the above:

L ≡ length (including wavelength)
f ≡ frequency
t ≡ time
m ≡ mass

and the subscript "$_r$" signifies the value for v=0, that is for when the two systems are at rest relative to each other.]

(The actual Lorentz Transforms, from which the above equations are derived, translate space-time coordinates from one system to the other. As with other aspects of traditional 20th Century physics the Lorentz Transforms and Contractions are intended to describe what happens but not how or why it happens.)

The only mode of behavior of the centers-of-oscillation that is consistent with all of the foregoing requirements is as follows.

CENTER-OF-OSCILLATION "RELATIVISTIC" BEHAVIOR

In order to describe the behavior of the center and the various differences in the propagated waves in different directions from the center, the propagation will be modeled as being resolved into three components: forward, rearward, and sideward. The directions are all relative to the direction of the center's velocity as depicted in Figure 13-2, on the following page. For purposes of analysis these orthogonal components represent the propagated wave in all directions. The wave in any particular direction is the resultant of that directions' projection on the forward or rearward component (whichever is at a nearer angle) and on the sideward component. ("Resultant" is the hypotenuse of the right triangle having the projection components as its other two sides.)

For a center at rest propagation of waves is the same in all directions at velocity $c = \lambda_r \cdot f_r$. See Figure 13-2 on the following page. (In the figure the "up", "down", "left" and "right" are all "sideward".)

A Center-of-Oscillation at Rest

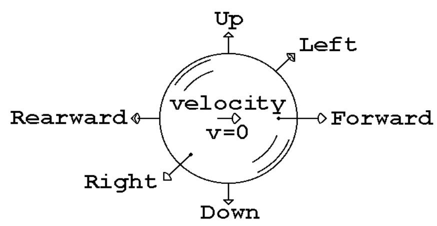

Figure 13-2

The effects that occur when this center is moving at velocity v are described in two steps.

Step #1

The center's rest frequency decreases and its rest wavelength correspondingly increases, the product still being c.

(13-5) $$f_v = f_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$$ [Center frequency decreases]

(13-6) $$\lambda_v = \lambda_r \cdot \frac{1}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$$ [Center wavelength increases]

(13-7) $$\lambda_v \cdot f_v = \lambda_r \cdot f_r = c$$ [Wave velocity still at c]

This also has the effect of decreasing the amount of rest mass since *mass ∝ frequency ∝ 1/wavelength*. Step #1 is the first of two steps of analysis of an actual event that occurs as one whole, therefore this decreased rest mass is an interim value in the development.

(13-8) $$m'_r = m_r \cdot \left[\frac{f_v}{f_r}\right] = m_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$$

A Center-of-Oscillation at Step #1

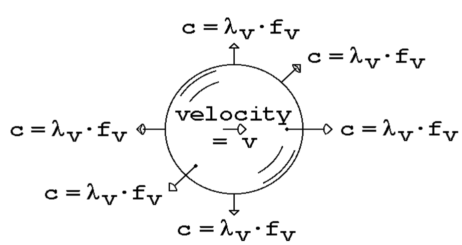

Figure 13-3

This effect occurs because of the limitation of the wave velocity in the medium to the speed of light, c. For a center at rest the propagated wave moves out of the way at velocity c. A center moving at velocity v finds (in the forward direction) the propagated wave not moving out of the way in time for the next cycle to begin as set by the at rest frequency of the center. The result is an imperative to reduce the center frequency by the factor $[1-{}^{v}/c]$. In the rearward direction the opposite is the case, an imperative to increase the center frequency by the factor $[1+{}^{v}/c]$. The center cannot do both at the same time. It can only oscillate at one specific frequency and so it responds "as best it can" by adopting a change in frequency by the geometric mean of the two conflicting factors as in equation 13-5.

Step #2

While the center can oscillate at only one frequency, it can propagate at different wavelengths in different directions. To maintain propagated wave velocity at c in the direction of center motion the wave must be actually propagated forward by the center at $c'=c-v$ relative to the center itself so that the wave velocity relative to at rest is the propagated velocity, c', plus the center velocity, v, that is $(c-v)+v=c$. To propagate forward at c' while maintaining the frequency at f_v requires that the wavelength change to a smaller value, λ_{fwd}. Likewise, rearward the wave must be actually propagated by the center at $c''=c+v$ relative to the center with a greater wavelength, λ_{rwd}.

The Wave as Propagated by the Center at Velocity v
(relative to the center)

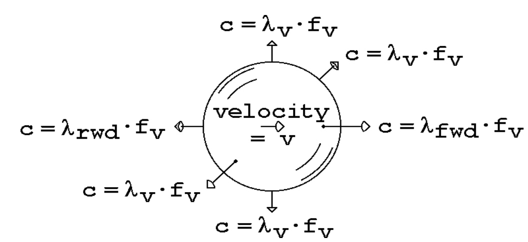

Figure 13-4

As the center "sees" it, per the above Figure 13-4, it is oscillating at f_v, with the wavelength (as is always the case) being set by the propagation conditions of the medium in which the wave travels in each direction. As "at rest" would "see" it, per Figure 13-5, below, the center appears to propagate different forward and rearward frequencies, f_{fwd} and f_{rwd}.

The Above Propagation as Observed from at Rest

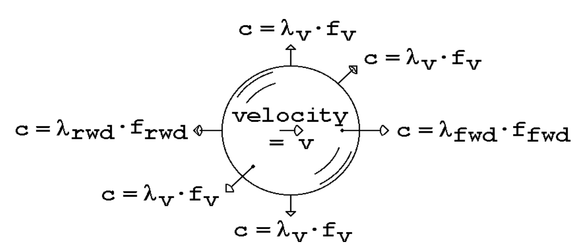

Figure 13-5

In the preceding figures the "*rwd*" (rearward) and "*fwd*" (forward) wavelengths and frequencies are as follows.

(13-9)
$$\lambda_{fwd} = \frac{c'}{f_v} = \frac{c-v}{f_v} = \frac{c\cdot\left[1 - \frac{v}{c}\right]}{f_r\cdot\left[1 - \frac{v^2}{c^2}\right]^{1/2}} = \lambda_r\cdot\frac{\left[1 - \frac{v}{c}\right]^{1/2}}{\left[1 + \frac{v}{c}\right]^{1/2}}$$
$$= \lambda_r\cdot\left[\frac{c-v}{c+v}\right]^{1/2}$$

(13-10)
$$f_{fwd} = \frac{c}{\lambda_{fwd}} = f_r\cdot\left[\frac{c+v}{c-v}\right]^{1/2}$$

(13-11)
$$\lambda_{rwd} = \lambda_r\cdot\left[\frac{c+v}{c-v}\right]^{1/2}$$

(13-12)
$$f_{rwd} = f_r\cdot\left[\frac{c-v}{c+v}\right]^{1/2}$$

Thus the field of propagated waves is traveling at c in all directions as observed by the center that is in motion and doing the propagating and as observed from at rest. The wave field also exhibits in all directions the relativistic Doppler effect, as it should.

So far the development demonstrates a decrease in rest mass, perhaps more properly referred to as a decrease in that part of the mass effect due to the overall frequency of oscillation of the center. In fact, the total mass increases by the same factor so that

(13-13)
$$m_v = m_r\cdot\frac{1}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} \qquad \text{[Total mass increases]}$$

To analyze how this occurs requires returning to the details of interaction of arriving wave potential impulse and center responsiveness.

The analysis here is of those factors affecting the interaction relative to Step #1, above, having already occurred that is, the starting point is the center with reduced rest mass, m'_r, reduced oscillation frequency, f_v, and increased wavelength, λ_v (equations 13-8, 13-5 and 13-6). The procedure is to investigate the change in the center's responsiveness due to its motion, the change as viewed from the direction of each of the orthogonal components of the oscillation.

Referring back to equation 12-3 and equation 12-24 of the prior section, the wave quantity of equation 12-3 is equation 12-24.

(12-3) Acceleration = Wave · Responsiveness, (12-24) $$\text{Wave} = U_S \cdot c$$

But now at velocity v, in the forward direction the wave is propagated by the moving center at $c' = c - v$ so that the propagation velocity in 12-24 is changed by the factor $c-v/c$, a reduction.

(13-15) $$\frac{c'}{c} = \frac{c-v}{c} = 1 - \frac{v}{c}$$

Since the propagated wave is reduced by that factor in the forward direction, the effective amplitude of the center in that direction must also be so reduced. That is a change in the second of the three factors in responsiveness (equation 12-8): cross-section, amplitude and repetition rate.

Furthermore, because the center is now moving at velocity v, in the forward direction, toward incoming waves from a source center, the repetition rate, the third of the three factors in the equation 12-8 expression for responsiveness of wave center interaction, is increased. That is it has increased by $c+v/c$.

(13-16) $$\frac{c+v}{c} = 1 + \frac{v}{c}$$

The combination of these two factors changes the at rest responsiveness by a factor equal to the product of the above two factors, that is by

(13-17) $$\text{product of factors} = \left[1 - \frac{v}{c}\right] \cdot \left[1 + \frac{v}{c}\right]$$

$$= \text{change in responsiveness}$$

$$= 1 - \frac{v^2}{c^2}$$

a reduction in responsiveness, an increase in mass.

Exactly analogous reasoning for the rearward direction results in the same overall change factor. Equation 13-15 now becomes (for rearward)

(13-18) $$\frac{c'}{c} = \frac{c+v}{c} = 1 + \frac{v}{c}$$

and equation 13-16 now becomes (for rearward)

(13-19) $$\frac{c-v}{c} = 1 - \frac{v}{c}$$

and the product of the two is the same equation 13-17.

For the direction of each of the four components to the side (up, down, right, and left) the case of interaction with waves coming in at right angles to the direction of motion of the center, the repetition rate is unchanged. There is no motion of the center toward or away from the incoming waves so there is no factor for a change in repetition rate due to such an effect. (The repetition rate is less due to the reduction of the center's frequency from f_r to f_v at Step #1, equation 13-5, but that has already been accounted for in that step and the current changes are relative to the results of Step #1.) The cross-section is no longer a circle, however. In the forward direction the at rest circle's radius has become λ_{fwd} instead of λ_v and in the rearward direction λ_{rwd} instead of λ_v.

The change factors are: forward

(13-20)

$$\lambda_{fwd} = \frac{c \cdot \left[1 - \frac{v}{c}\right]}{f_v} \qquad \text{[From equation 13-9]}$$

so that

$$\frac{\lambda_{fwd}}{\lambda_v} = \left[1 - \frac{v}{c}\right] \qquad [c = f \cdot \lambda]$$

and analogously rearward

(13-21)

$$\lambda_{rwd} = \frac{c \cdot \left[1 + \frac{v}{c}\right]}{f_v}$$

$$\frac{\lambda_{rwd}}{\lambda_v} = \left[1 + \frac{v}{c}\right]$$

so that the product of the change factors is, once again, equation 13-17.

Thus from every direction the Step #2 responsiveness is reduced by the factor of equation 13-17 due to the motion of the center at velocity v. The mass is accordingly increased by the reciprocal of that factor. These changes are relative to the conditions at the end of Step #1, where the mass of the center was as in equation 13-8. Therefore m_v, the center overall mass at velocity v, is

(13-22)

$$m_v = \begin{bmatrix}\text{Mass } m'_r \text{ of} \\ \text{equation} \\ 13\text{-}8\end{bmatrix} \times \left[\frac{1}{\text{Factor of equation 13-17}}\right]$$

$$= \left[m_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}\right] \times \left[\frac{1}{1 - \frac{v^2}{c^2}}\right]$$

$$= m_r \cdot \frac{1}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$$

which is the same as that called for by the Lorentz Contraction in equation 13-4.

MASS AND ENERGY RELATIONSHIPS DUE TO MOTION

The total mass, m_v, of the center when at velocity v, can be described as the sum of the modified rest mass of Step #1 above, m'_r, per equation 13-8 plus whatever increase resulted from Step #2, above, here now designated m_k.

(13-23) $$m_v = m'_r + m_k$$

Expressing m_v and m'_r in terms of m_r, the original rest mass, using equations 13-8 and 13-20, above, equation 13-23 becomes equation 13-24, below.

(13-24) $$\frac{m_r}{\left[1 - \frac{v^2}{c^2}\right]^{½}} = m_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½} + m_k$$

[Substitute into equation 13-23]

$$m_r = m_r \cdot \left[1 - \frac{v^2}{c^2}\right] + m_k \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$$

$$m_r \cdot \left[\frac{v^2}{c^2}\right] = m_k \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$$

[Simplify]

$$m_k = \frac{m_r \cdot \left[\frac{v^2}{c^2}\right]}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$$

[Solve for m_k]

$$= m_v \cdot \left[\frac{v^2}{c^2}\right]$$

From equation 13-22 $m'_r = m_v \cdot$*[Factor of equation 13-17]*. Equation 13-23 can be restated by substituting that and the above equation 13-24 for m_k into equation 13-23 to obtain equation 13-25, below.

(13-25) $$m_v = m'_r + m_k$$

$$= m_v \cdot \left[1 - \frac{v^2}{c^2}\right] + m_v \cdot \left[\frac{v^2}{c^2}\right]$$

[Total Mass] = [Mass in Rest Form] + [Mass in Kinetic Form]

This relationship can be expressed in terms of energy by multiplying each of the terms (masses) by c^2. The result is equation 13-26 on the next page. The detail of the equation is pursued to make clear the distinction between Energy in Rest Form and Energy in Kinetic Form on the one hand versus Rest Energy and Kinetic Energy on the other hand. The latter, Rest Energy and Kinetic Energy, are the traditional terminology of 20th Century physics. While they are a useful point of view on the every-day macroscopic level, they do not correctly reflect the underlying reality that Energy in Rest and Kinetic Form reflect.

$$
\begin{aligned}
\text{(13-26)}\quad m_v \cdot c^2 &= m'_r \cdot c^2 + m_k \cdot c^2 \\
&= m_v \cdot [c^2 - v^2] + m_v \cdot v^2 \\
\begin{bmatrix}\text{Total}\\ \text{Energy}\end{bmatrix} &= \begin{bmatrix}\text{Energy in}\\ \text{Rest Form}\end{bmatrix} + \begin{bmatrix}\text{Energy in}\\ \text{Kinetic Form}\end{bmatrix} \\
&= \begin{bmatrix}\text{Energy in}\\ \text{Rest Form}\end{bmatrix} + \begin{bmatrix}\text{Rest Energy}\\ \text{Lost Step 1}\end{bmatrix} + \begin{matrix}\text{"Kinetic}\\ \text{Energy"}\end{matrix}\Bigg] \\
&= \begin{bmatrix}\text{Energy in} & & \text{Rest Energy}\\ \text{Rest Form} & + & \text{Lost Step 1}\end{bmatrix} + \begin{matrix}\text{"Kinetic}\\ \text{Energy"}\end{matrix} \\
&= \begin{bmatrix}\quad\text{Rest Energy}\quad\end{bmatrix} + \begin{matrix}\text{"Kinetic}\\ \text{Energy"}\end{matrix}
\end{aligned}
$$

The new concepts are termed Energy-in-Rest-Form and Energy-in-Kinetic-Form because they correspond to the behavior of the center-of-oscillation at velocity, v. Of the changes that take place when a center changes from being at rest to at velocity, v, at Step #1 per Figure 13-3 the center is still symmetrical in form, that is its frequency and wavelength are the same in all directions. The center's general form is the same as a center at rest except for decreased frequency, f_v, and correspondingly increased wavelength λ_v.

At Step #2, Figure 13-4, the center's behavior changes in one way forward and in the opposite way rearward. The kinetic effect makes the form of the center unsymmetrical like an arrow pointing in the direction of motion. The portion of the total energy due to Step #1 is Energy in Rest Form. The additional energy due to Step #2 is Energy in Kinetic Form.

Not only does this new point of view, Energy in Kinetic Form, better correspond to the underlying physical reality; it will also be seen to be fundamentally important when matter waves are taken up in section *15 - Quanta and the Atom*. The traditional relativistic derivation of kinetic energy also directly yields this result, Energy in Kinetic Form, but it was not recognized in the development of 20th Century physics. See detail dotes *DN 4 - Derivation of Mass, Energy in Rest, Kinetic Form*, following this section.

In addition to the velocity effects of mass increase and frequency decrease (and time slowing), there is also the contraction of length in the direction of motion. The contraction of center cross-section has already been developed, that is, the cross-sectional "diameter" in the direction of motion is modified by motion as described above for Step #2 by the factor of equation 13-17 relative to λ_v. Since λ_v is increased in Step #1 from λ_r per equation 13-6 the overall change in center "diameter" in the direction of motion is as expected from the Lorentz Contraction for length, per equation 13-1.

That contraction of the center itself is only of minor significance in terms of the Lorentz Transform, however, because the physics of the Transform is not aware of centers. In any case, the center contraction's only effect is that on mass and responsiveness which have already been treated. However, the contraction on the macroscopic scale must yet be investigated.

On the macroscopic scale it is necessary to investigate two centers and the distance between them in order to develop a motion-caused contraction of length in matter. In bulk matter composed of multiple particles, atoms and their components, the spacing of the atoms depends on the balance of the various electrostatic forces acting as a result of the centers-of-oscillation, protons and electrons, of which the matter atoms are composed. Considering just two centers-of-oscillation at rest in a fixed position relative to each other, the effect of their moving jointly at velocity v in the direction of the line joining them should be a closer spacing of the two centers by the Lorentz Contraction factor.

The position of each of the two centers is the balance of all of the forces acting on the centers, an equilibrium position. If the motion is to change the distance between the two centers then the force acting between the two centers must change so that a new closer equilibrium spacing exists and determines the new distance between the two centers. For the centers to need to be closer in order to re-establish equilibrium the effective charge of each of the centers must have decreased.

In other words, for the Coulomb force

(13-27)
$$F = \frac{Q_1 \cdot Q_2}{d^2}$$

to be unchanged even though d is reduced by the Lorentz Contraction (per equation 13-1) by the factor

(13-28)
$$\frac{d_v}{d_r} = \left[1 - \frac{v^2}{c^2}\right]^{½}$$

so that d^2 is changed by the factor

(13-29)
$$\frac{d_v^2}{d_r^2} = \left[1 - \frac{v^2}{c^2}\right]$$

then $Q_1 \cdot Q_2$ must be so reduced by the same factor as is d^2.

But, that is exactly the case. It has already been shown that the forward wave and center amplitude is reduced by the factor $[1-{}^{v}/c]$ because of the forward propagation at $c' = c - v$ and that the rearward wave and center amplitude is analogously changed by the factor $[1+{}^{v}/c]$. The forward wave of the trailing center interacting with the rearward component of the leading center's oscillation amplitude is thus changed by the product of these two factors. Likewise the rearward wave of the leading center interaction with the forward component of the trailing center's oscillation is changed by the product of the factors.

The product of the factors is (equation 13-17, again)

(13-17)
$$\text{product of factors} = \left[1 - \frac{v}{c}\right] \cdot \left[1 + \frac{v}{c}\right]$$
$$= 1 - \frac{v^2}{c^2}$$

so that the effective $Q_1 \cdot Q_2$ is reduced by that amount and the distance between the two centers, d must shorten so that d^2 is reduced by that amount, a reduction of the distance between the two centers by the Lorentz contraction amount, the square root of the equation 13-17 factor.

This contraction of inter-center spacing in the direction of motion produces the macroscopic Lorentz contraction of length in the direction of motion in matter.

There is, however, another component to the interaction. While, in the forward direction, the source center propagates the wave at $c'=c-v$, the wave actually travels at velocity c because it is propagated by the center itself traveling forward at v yielding the overall wave velocity as $c'+v = (c-v)+v = c$. The forward wave, attenuated as above by its propagation at c', is thus also "thrown forward" by the center's velocity. This adds another component of force, of potential impulse per wave times the wave repetition rate, that the wave can deliver to an encountered center.

In fact, without the wave having that additional component of force, and the consequent reaction back on the center in that same additional amount, the center would not experience equal reaction back on it in all directions from its propagated wave. The magnitude of this "force component" due to the center's momentum or its velocity is $[^V/c] \cdot F_r$, where F_r is the force that the wave would deliver if at rest and which it does always deliver to the sides: up, down, right and left.

Likewise for the rearward wave, the wave is "negatively" "thrown out" in the same amount, otherwise the reaction back on the center by the rearward propagated wave would be $[^V/c] \cdot F_r$ greater than the rest case. Without the "force component" the center would be self-accelerated by a force of $2 \cdot [^V/c] \cdot F_r$ in the forward direction (the forward and rearward effects combined), clearly not the case and unacceptable.

Returning to the case of two centers traveling in the direction of an imaginary line joining them, when the forward wave of the trailing center encounters the rear of the leading center (which is traveling at the same velocity as the trailing center as part of the overall piece of matter being considered) the $+[^V/c] \cdot F_r$ positive "force component" of the forward wave and the $-[^V/c] \cdot F_r$ negative "force component" of the rear of the encountered leading center cancel out leaving the net action due to the encounter as presented above before considering the "force component due to center velocity or momentum" aspect. The situation is the same with the rearward propagated wave of the leading center encountering the front of the trailing center. The net effect on the interaction is null, but the phenomena are still there.

RELATIVITY AND INVARIANCE

Leaving the above subject of the behavior of a center in motion for a moment, it is necessary here to review briefly the history of the development of the Theory of Relativity.

By the time of Newton and the development of his laws of motion it was well understood that all motion is relative to some frame of reference. One cannot say that something is moving at a stated velocity except by stating what that velocity is relative to. Newtonian mechanics dealt with this problem, successfully for "Newtonian systems". Simple direct relationships exist to transfer Newtonian motion descriptions from one frame of reference to another, e.g., how a flying bird appears to a person standing on the ground versus a person who is riding on a train at 30 miles per hour.

In the second half of the 19th century Maxwell developed his equations describing electromagnetic field, the equations being an outgrowth of the then developing understanding of electricity, charge, magnetic effects, etc. Substantially before the first actual detection of electromagnetic waves by Herz toward the end of the century, it was recognized that Maxwell's equations described a wave propagating in space at a velocity, c, determined by two constants in the equations, ε and μ, the dielectric constant and the permeability of whatever medium the waves were passing through, such that $c^2 = 1/\mu \cdot \varepsilon$. But, this result presented two problems.

First

> At the time it seemed inconceivable that these (or any) waves could propagate other than in some medium. Since the waves could and do propagate throughout free space as well as through the air and through other substances some kind of all-pervading medium, called in those days an "aether", was postulated.

Second

> Maxwell's equations would not correctly transform from one frame of reference to another at different velocity using the Newtonian transformations. Thus it was assumed that Maxwell's equations applied only to one, prime, frame of reference, that of the "aether", which also defined ε, μ and, therefore c.
>
> [The Newtonian transform between two systems at different velocities is to merely subtract the velocity difference. For example, to a passenger in a train going forward at 30 miles per hour the train is a stationary reference system and the landscape out the window is traveling backwards at 30 miles per hour. To do a Newtonian transform from the train-as-reference to the landscape-as-reference one subtracts the landscape's 30 miles per hour backward from the landscape (making it stationary) and also from the train (making it be going 30 miles per hour forward).]
>
> [If one attempts such a Newtonian transform on Maxwell's equations and the speed of light wrong results are obtained. One cannot subtract a velocity difference between two systems from the speed of light, c, because c is an absolute constant given by $c^2 = 1/\mu \cdot \varepsilon$ not variable with some other velocity.]

The problem in these assumptions was that all attempts to define and detect the "aether" led to contradictions or further problems. The most famous of these attempts was the Michaelson-Moreley experiment, which, expecting to find two different measured results for the speed of light because of the motion of the earth in its orbit relative to the "aether" obtained the "negative" result that the speed of light always measured to be the same regardless of the motion of the observers, Michaelson and Morely and the Earth.

The Michaelson-Moreley experiment and the Newtonian transformation problem required that a new transformation system be developed, which was done primarily by Lorentz. Lorentz retained the existence of an "aether" which had to be the prime frame of reference. His transformations and their consequent "contractions", presented earlier in this section, resolved the problems

In the early 1900's Einstein took the further step of denying that any "aether" or medium was necessary for electromagnetic waves and that there was no prime frame of reference. These concepts were embodied in his Theory of Relativity for which there is no "aether" and in which everything is defined to be relative.

Analogously to the manner in which earlier failures to develop a theory of how electric charge produces an "action at a distance" led to defining the electric "field" as the conveyor of the action and the then abandonment of any further investigation of the issue; so, also, the repeated failure to successfully define and detect an "aether", coupled with Einstein's formulation that dealt with the problem by denying the "aether's" existence, resulted in the complete acceptance of Einstein's theories and the abandonment of the "aether" problem.

Excepting only the issue of whether some kind of "aether" exists and is the prime frame of reference, the Lorentz and the Einstein formulations are equally valid descriptions of physical reality. However, the Theory of Relativity and other developments in physics that came from Einstein were tremendously successful. The problem of invariance and the behavior of electromagnetic waves appeared to be resolved. Relativistic effects could be observed and measured experimentally in the laboratory. The mass-energy equivalence was dramatically confirmed.

Just as Einstein had his doubts about some of the well accepted aspects of traditional 20th Century physics (in referring to some aspects of uncertainty and quantum mechanics he is reputed to have said that he "... did not believe that God plays with dice ....") so Lorentz clung to the necessity of an "aether" and the prime frame of reference that it implied.

But the relativity "bandwagon" was rolling and relativity carried the day.

Returning, now, to the subject of the center-of-oscillation in motion, what has actually been described so far is the behavior of a center due to motion at constant velocity, v, relative to wherever the center last was when it was "truly" at rest, that is when the circumstances were such that none of the described consequences of motion were occurring and the center's state was as set out in Figure 13-2, which is to say when the center was at rest relative to the medium.

All of this is to say that there must be an absolute frame of reference to which all motion is relative. Whereas the issue of the existence or non-existence of an "aether" could be abandoned without problem at the time of Einstein and Lorentz, it now must be addressed; and, since a medium, one that functions as a prime frame of reference, is now necessary it turns out that the solution of Lorentz was correct and that of Einstein was wrong.

It is now necessary to restate relativity more correctly. There is nothing inherent in Einstein's Theory of Relativity requiring absolute relativity, the absence of a prime frame of reference. The concept "relative" does not necessarily enter into the mathematical derivations and "theory of relativity" is a misnomer. The theory-system called the Theory of Relativity should be correctly referred to as the "Principle of Invariance". Einstein's postulates were solely invariance.

"Invariance" means that the laws of physics, the behavior of all physical reality, is the same in any coordinate system or frame of reference. Invariance requires that the <u>form</u> of the mathematical statements describing reality and the <u>constants</u> appearing in those statements be invariant under any transformation of coordinates, which means that they must be unchanged by any change of frame of reference regardless of its motion so long as it is at constant velocity with no acceleration involved. Since all universal constants appearing in equations describing physical reality are invariant, the speed of light, one of those constants, is invariant.

Invariance includes the principle that the "interval", the space-time "distance", between two events is invariant regardless of the frame of reference or coordinate system. This interval is defined by the relationship

$$\text{(13-30)}\quad \text{Interval}_{space}{}^2 = [c\cdot\delta t]^2 - [\delta x^2 + \delta y^2 + \delta z^2]$$

or

$$\text{Interval}_{time}{}^2 = \delta t^2 - [\delta x^2 + \delta y^2 + \delta z^2]\cdot\frac{1}{c^2}$$

where, given one event at (t_1, x_1, y_1, z_1)
and a second event at (t_2, x_2, y_2, z_2),
then

δt = time between the events

δx, δy, δz = three dimensional distances between the events.

The principle of invariance is not magical or mysterious, but obvious. When one walks down the street, breathes, throws a stone or rides in a space ship one is doing <u>*a*</u> thing. The thing is not changed by changing the frame of reference from which someone observes it. The act is invariant therefore its description must be so.

To be perfectly clear about this replacement of relativity with "absolutivity" the pertinent factors are as follows.

(1) All motion is absolute, that is, it is relative to an absolute, prime frame of reference.

> In normal human experience the absolute frame of reference cannot be detected so that motion seems to be relative, but that is only an appearance.

(2) The absolute frame of reference is not a "preferred" frame of reference in the sense of having special or different physical laws. It is a "prime" reference system in that all physical reality is relative to it.

> That is why the universe is invariant. For physical reality there is only one grand system of reference

for everything. The universe does not "know" about our frames of reference; it simply is in its natural frame of reference everywhere. It would be ridiculous for it not to be invariant.

(3) The absolute frame of reference is the U-wave medium, both $+U$ and $-U$, in which U-waves and centers-of-oscillation exist; that which came into existence at the beginning as described in earlier sections of this work.

(4) The Einsteinian, or geometrodynamic, theory of space-time and gravitation is still valid. The Lorentz transforms and contractions are still valid. Now, however, absolutivity and the universe description developed here constitute a simple underlying reality for those phenomena.

This contention goes counter to some of the most basic accepted concepts of 20th Century physics. Consequently, it requires substantial justification, which is as follows (starting with the weaker arguments).

(1) Lorentz showed that a medium not participating in the motion of anything is consistent with all data including the Michaelson-Moreley and similar experiments.

(2) The absolute velocity of the Earth (detailed in (5), below) is sufficiently low that observations from Earth are equivalent (within the accuracy involved) to observations from at rest in the absolute frame of reference. (Symbol "≈" means "approximately equals").

(13-31)

$$v_{Earth} \approx 370\ {}^{km}/sec$$

$$\left[1 - \frac{v_E{}^2}{c^2}\right]^{½} = 0.9999992...$$

(3) A medium is required for electromagnetic waves. They either propagate in a medium or are themselves propagation of the wave "substance" or else they have no existence. Since they exist, and since their propagation is a transverse wave, and since there has never been a contention that electromagnetic waves involve motion of anything in the direction of wave propagation other than that of the wave's energy and momentum, the medium must exist.

(One cannot say that there is no E-M wave medium just "field". It has already been pointed out that "field" is merely a "code-word" for "action at a distance" having no meaning otherwise. In any case, the field has now been shown to be the U-wave propagation in the medium.)

A medium is also required to define and set the propagation velocity of the waves to c, the speed of light. Without a medium there is no cause of a universal fixed value of c nor μ and ε, the dielectric constant and permeability of free space.

(4) As described in the General Theory of Relativity, "curved" space-time , which is due to the variation of gravitation with the distribution of mass in the universe, and the gravitational field pervading the universe with its shape due to that variation, is itself a frame of reference. Since space-time is not uniformly "flat", the shape variations make possible detection not only of acceleration but also of absolute velocity relative to the total mass as distributed in the universe.

But, that reference frame is identical to the reference frame of the singularity (single point) at which the universe started, and, therefore is identical to the U-wave medium as prime frame of reference.

It was stated at the beginning of the prior section that "... all field is one aspect or another of the waves propagated by centers-of-oscillation ...". That statement also applies to gravitational field as is developed in section *19 - Gravitation*. Thus the gravitational field "framework" of space is actually a U-wave / medium framework.

(5) There exists throughout the universe a background radiation which is the residual radiation from the immense energy of the "big bang", the start of the universe. The temperature has now cooled down from the extremely high levels at the beginning to only about 2.7° *Kelvin* (above absolute zero). This radiation is, of course, relative to the beginning, relative to the U-wave medium. Measurements of Doppler frequency shift of this radiation due to the motion of the Earth give an absolute velocity for the Earth relative to the medium of about $370\ ^{km}/sec$, as was used in (2) above. The direction of the Earth's motion as indicated by those measurements is off in the direction from Earth of the constellation Leo.

(6) The Lorentz contractions must actually occur, not be mere observational effects. According to relativity, an object in motion experiences slower time. If two identical clocks agree and one clock is then moved away and returned while the other is motionless (in relativistic terminology if one is moved away and then returned relative to the other from which observations are made) the moved clock must read an earlier time than the unmoved clock even when both are again at rest in the same frame of reference. When both are so again together and at rest there can be no observational quirk to cause them to read different times. The moved clock must have actually run slower.

(It could be argued that the moved clock had to be accelerated to be moved so that the overall process was not a constant velocity situation. That is not the contention of relativity, however, which states that the moved clock does run slower and relies on the fact of acceleration to make the distinction as to which clock was moved and which stayed at rest.)

(7) Consider three clocks, #1, #2, and #3, each at some constant velocity relative to each of the other two. Clock #1 observes the time of clock #3 contracted by some amount. Clock #2 likewise observes Clock #3 with time contracted but by some different amount than Clock #1 saw. But, Clock #3, which is observed with a time contraction by Clock #1 in an amount based on the velocity difference between Clock #1 and Clock #3, and which is also observed by Clock #2 with a time contraction based on the velocity difference between Clock #2 and Clock #3 cannot be actually contracted two different amounts at the same moment. Since the contraction must be actual, not solely observational, relativity has an absurdity here.

The solution to this last problem is simple. All clocks are actually (which is as observed from the prime frame of reference) contracted according to their absolute velocity relative to that frame, the medium, not according to their velocity relative to another moving clock. In addition, an observer at a moving clock observes somewhat different results than those actual contractions because his standards of measurement have also been contracted by his motion (even though they appear unchanged to him). This produces an observed, but not actual modification of the absolute, actual contraction.

(Of course, if one of the moving clocks is moving at a modest velocity the difference between its at rest dimensions and its actual contracted ones is so small that the observations from that slow-moving clock would be essentially equivalent to from at rest the very case set out for planet Earth in (2) above.)

In his original paper on relativity Einstein contended that there was no way that an observer experiencing acceleration could distinguish between whether his system was actually accelerating in a region free from gravitation or was actually at rest in a gravitational field. In fact, that contention is incorrect and the distinction can be made by local measurement, as is now known. The distinction occurs because gravitation follows an inverse square law in practice in the real universe.

One could say that Einstein was largely correct but for partially incorrect reasons. The same can be said of the effect of absolutivity on cosmology and space-time physics. The results obtained by traditional 20th Century physics and the theories leading to them are largely correct. Absolutivity only restores the medium and the prime frame of reference.

The fact that until recently we could detect no absolute velocity and that even now it is only detectable with special scientific effort does not mean that all motion is relative, it only means that we have not developed the means for ready detection of absolutivity. There have been many other things that were undetectable in the past but that are not so now: germs, distant stars, x-rays, atoms, etc.

The Theory of Relativity has required mind-twisting adjustments to way of thinking, adjustments away from the reasonable and "apparent" to a mass of paradoxes and their resolutions. Absolutivity retains contact with reality both in describing physical reality accurately and by doing so in a fashion much more consistent with reasonableness.

With absolutivity the principle of invariance becomes simple, practical and apparent in addition to being necessary for science as it always was. There is only one "system", the universe with some parts moving in various ways and some parts at rest and that one system has, of course, one overall set of physical laws throughout. Before absolutivity, invariance was necessary but was crying for an explanation. One can see no particular reason why invariance should be necessarily automatically true in the universe of the Theory of Relativity. Absolutivity solves the problem by showing the natural inevitability of invariance.

Why does this new medium, the U-wave medium, succeed when all prior attempts to define an "aether" without contradictions failed ? This medium exists yet it overall cancels out to null between *+U* and *-U*. However, in their gravitational field aspect U-waves pervade the universe although generally at greatly reduced amplitude due to the inverse square law effect being part of the nature of the gravity aspect of the U-waves. Thus the medium is identical to the "field" of traditional 20th Century physics, of Einstein's theories.

(Gravitation is treated fully in section *19 - Gravitation*. The nature of the U-waves and their propagation is treated fully in section *21 - The Probable End*. It is shown there that the medium <u>is</u> the propagating U-waves.)

It is now time to address the apparent paradox that was left as a question toward the end of section *11 - Electric Field and Charge*. The apparent paradox had two elements.

First

> A charge at rest relative to the Earth's surface exhibits to us, who are also at rest relative to the Earth's surface, no magnetic field even though the charge is clearly in motion with the Earth's surface rotating about the planet's axis, revolving about the sun and moving relative to and with the galaxy.

Second

> A charge in motion in an electric wire (as a current) does exhibit a magnetic field to us, who are (in this problem) moving with the same velocity as the charge) even though the charge is at rest relative to us.

Although there are these two elements to the problem, they are one overall problem, an apparent inconsistency in physical laws. The inconsistency results directly from relativity and resolves when absolutivity is applied.

Considering first the problem of the wire, absolutivity answers with the solution,

> "Since the current in the wire is in absolute motion, it exhibits the usual magnetic field regardless of the motion of the observer. The only effect of the observer's motion is to change his standards of measurement and, therefore, the magnitude of the magnetic field as he measures it."

Relativity responds,

> "No, the explanation is that, although the current of the charge moving relative to the wire is zero relative to the observer

> moving at the same velocity, the overall wire including the charge is electrically neutral so that the wire moving 'rearward' without the charge (as the observer sees it) is an opposite charged wire moving in the opposite direction and produces the same magnetic field to the observer as he would see if he were at rest relative to the wire and he were observing the charge moving 'forward'. In other words, a wire moving 'rearward' while its current stands still gives the same magnetic field as the wire standing still and its current moving 'forward'."

Absolutivity then closes the discussion with,

> "If relativity were valid that would be a true and good analysis, but the same problem as that of the wire can be stated for a beam of charged particles in empty space without the wire. In such a case the magnetic field behavior is the same, the paradox for relativity is the same, but there is no 'wire' to travel 'rearward'. Thus, only the explanation of absolutivity will resolve the problem."

(This also illustrates the simplicity of absolutivity as compared to the complications of relativity.)

The first part of the paradox, that of the charge at rest on the Earth's surface, is simply a case of magnitudes. In fact the charge at rest relative to the moving Earth is in absolute motion and does exhibit the expected magnetic field. However, the field is too small to be noticed. As is developed in the next section, "Magnetic and Electromagnetic Field", the magnitude of magnetic field is less than the corresponding electric field magnitude by a factor of $[v^2/c^2]$. The velocity of Earth (presented earlier above) is less than 10^{-3} of the speed of light so that $[v^2/c^2] < 10^{-6}$.

Footnote 13-1

The conceptual model as so far presented of the medium, centers-of-oscillation, and the propagated waves is of a static medium in which center oscillations and waves take place. For the moment there is no harm in that conception in that it produces no errors to this point. However, the medium has a much more active role, which is further developed in Section 16, then in Section 19, and finally in Section 21.

DETAIL NOTES 4

Derivation of Mass, Energy in Rest, Kinetic Form

The traditional relativistic derivation of kinetic energy is as below. This derivation is also the demonstration that

$$\text{Energy} = \text{mass} \cdot c^2$$

(See Detail Notes *DN 1 - Differential Calculus, Derivatives* and *DN 5 - Integral Calculus (Mathematics of Summing Infinitesimals)* for an explanation of the differential and integral calculus involved.)

STEP 1 - EINSTEIN'S ORIGINAL DERIVATION

Kinetic energy, *KE*, is defined as the work done by the force, *f*, acting on the particle or object of mass, *m*, over the distance that the force acts, *s*. This quantity is calculated by integrating the action over differential distances.

(DN4-1) $$KE = \int_0^s f \cdot ds \qquad \text{[Per above definition]}$$

(DN4-2) $$= \int_0^s \frac{d(m \cdot v)}{dt} \cdot ds \qquad \text{[Newton's 2nd law]}$$

(DN4-3) $$= \int_0^{(m \cdot v)} \frac{ds}{dt} \cdot d(m \cdot v) \qquad \text{[Rearrangement of form]}$$

(DN4-4) $$= \int_0^{(m \cdot v)} v \cdot d(m \cdot v) \qquad [v = {}^{ds}/dt]$$

(DN4-5) $$= \int_0^v v \cdot d\left[\frac{m_r \cdot v}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}}\right] \qquad [m \text{ is } m_r \text{ Lorentz contracted by } v]$$

(DN4-6)

$$= \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{½}} - m_r \cdot \int_0^v \frac{v \cdot dv}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$$ [integration by parts]

(DN4-7)

$$= \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{½}} + m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]^{½} - m_r \cdot c^2$$ [Integration of 2nd term]

(DN4-8)

$$= \frac{m_r \cdot v^2 + m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]}{\left[1 - \frac{v^2}{c^2}\right]^{½}} - m_r \cdot c^2$$ [Rearrange-ment of form]

(DN4-9)

$$= \frac{m_r \cdot c^2}{\left[1 - \frac{v^2}{c^2}\right]^{½}} - m_r \cdot c^2$$ [Simplification]

(DN4-10) $KE = m_v \cdot c^2 - m_r \cdot c^2$ [Lorentz transform]

(DN4-11) {Kinetic Energy} = {Total Energy} - {Rest Energy}

or

{Total Energy} = {Kinetic Energy} + {Rest Energy}

The appearance in this result that the energies are the product of the masses times c^2, the speed of light squared, was the origination of that concept, the famous $E = m \cdot c^2$. The concept falls out naturally from applying the Lorentz transforms to the classical definition of kinetic energy. It is somewhat surprising that Einstein was the first to do that inasmuch as Lorentz developed the Lorentz transforms and contractions.

STEP 2 - ALTERNATIVE VARIATION ON STEP 1

If, in the above traditional derivation, one proceeds differently from equation DN4-7 on, as below, energy and mass in kinetic form and rest form result.

(DN4-7)

$$KE = \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{½}} + m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]^{½} - m_r \cdot c^2$$ [Repeated as start point]

(DN4-12)

$$KE + m_r \cdot c^2 = \frac{m_r}{\left[1 - \frac{v^2}{c^2}\right]^{½}} \cdot v^2 + m_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½} \cdot c^2$$ [Move $m_r \cdot c^2$]

Considering and evaluating the three terms of equation DN4-12:

$KE + m_r \cdot c^2$ = Kinetic plus rest energies
= Total Energy

$m_r \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$ = The reduced rest mass, m'_r, at the first of the two steps in a center's change from at rest to at velocity v (see equation 13-8)

= Mass in Rest Form which when multiplied by c^2 then gives the Energy in Rest Form

$\frac{m_r}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$ = The relativistically increased mass, m_v, at velocity v per equation 13-13. By default, in equation DN4-12 (i.e. by total energy less that in rest form) this mass times v^2 is the Energy in Kinetic Form.

the result is that equation DN4-12 is equivalent to

(DN4-13) [Total Energy] = [Energy in Kinetic Form] + [Energy in Rest Form]

$$m_v \cdot c^2 = m_v \cdot v^2 + m_v \cdot (c^2 - v^2)$$

which means that (dividing the above energy equation by c^2 to obtain an equation in mass)

(DN4-14) [Total Mass] = [Mass in Kinetic Form] + [Mass in Rest Form]

STEP 3 - RECONCILIATION

Classical, Newtonian non-relativistic kinetic energy is derived as follows. A mass, m, acted on by a constant force, F, over a distance, s, has potential energy, W_{pe}, at the start all of which becomes kinetic energy, W_{ke}, by the end. The force produces a constant acceleration, a, and an increasing velocity, v, as the distance, s, is traversed.

(DN4-15) $a = {}^{F}/m \qquad v = a \cdot t \qquad s = \int v \cdot dt = ½ \cdot a \cdot t^2$

$$W_{pe} = F \cdot s = \left[m \cdot a\right] \cdot \left[½ \cdot a \cdot t^2\right]$$

$$= ½ \cdot m \cdot \left[a^2 \cdot t^2\right]$$

$$= ½ \cdot m \cdot v^2 \qquad = W_{ke}$$

Why is the formulation for classical *Kinetic Energy* $KE = ½ \cdot m \cdot v^2$ but *Energy in Kinetic Form* is simply $m \cdot v^2$ without the ½ ? Energy in Kinetic Form, reflecting the actual changes in the oscillation frequency and propagation wavelengths of the center-of-oscillation, includes not only the total energy increase above the rest value but also the difference between that rest value and the reduced *Energy in Rest Form* at the current velocity.

When dealing with quite small velocities (v small relative to c) the excursion of total energy above rest energy and the excursion of energy in rest form below rest energy are both essentially linear. In that case the portion above the rest case is essentially half of the total excursion above and below the rest case. The classical kinetic energy is then half, $½ \cdot m \cdot v^2$, the total energy in kinetic form, $m \cdot v^2$, for $[{}^{v}/c]$ quite small.

Integral Calculus (Mathematics of Summing Infinitesimals)

(If not familiar with differential calculus then first see Detail Notes *DN 1 - Differential Calculus, Derivatives* first.)

Consider the problem of a force acting on an object causing it to accelerate (per Newton's second law) so that its velocity increases and, therefore, its energy increases. For example, when a compressed spring is released it exerts a force upon whatever it encounters in expanding back to its relaxed position. How much energy does the spring deliver ? While this problem is trivial, it can well illustrate the method for non-trivial situations.

The force that the spring exerts is proportional to the distance that the spring is compressed from its relaxed position.

(DN5-1) $f = F_0 - k \cdot s$

where: k = a constant characteristic of the spring
s = the displacement from the fully compressed position
F_0 = the fully compressed force of the spring

Graphically the relationship of equation DN5-1 is as in Figure DN5-1, below.

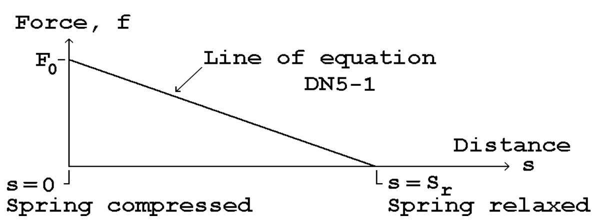

Figure DN5-1

The energy, w, delivered by the spring upon its release allowing it to return from position $s=0$ to $s=S_r$ is equal to the force times the distance through which the force acts. But, since the force continuously decreases as in the figure, how is the force times distance to be calculated ?

Referring to Figure DN5-2, below, for any particular value of s, for example $s=S_{sample}$ the portion of the action from s a little less than S_{sample} to s a little more than S_{sample} could be approximated by a thin rectangle for which we assume that from the left to the right side of the rectangle the force, f, is essentially constant at its value in the middle of the rectangle.

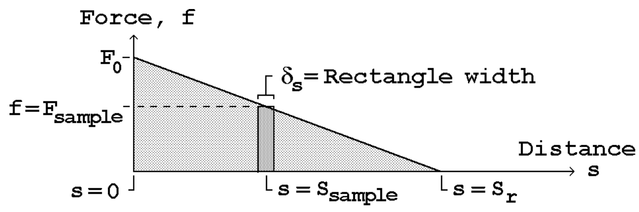

Figure DN5-2

The area of the rectangle, being force times distance, is energy. The rectangle's contribution of energy, δw, to the overall energy, w, is the rectangle's force, F_{sample}, times the rectangle's distance through which the force acts, δs,

(DN5-2) $\delta w = F_{sample} \cdot \delta s$

$= [F_0 - k \cdot S_{sample}] \cdot \delta s$ [Substitute for F_{sample} per equation DN5-1]

or, generalizing equation DN5-2 for any value of s over the range of the spring's action, that is for any such rectangle over the range, and letting the width, δs, of the rectangle become infinitesimal, ds, then

(DN5-3) $dw = [F_0 - k \cdot s] \cdot ds$

so that now the problem is one of how to sum up all of those individual infinitesimal rectangular contributions to get the total.

This type of problem occurs frequently in science and engineering. The formulation can be generalized to the form

(DN5-4) $dw = f(s) \cdot ds$

where dw is the infinitesimal increment of the result sought, ds is the infinitesimal increment of the independently variable quantity in the situation and $f(s)$ is a mathematical expression which is a function of s (its value is dependent on the value of s) and which relates s to the result sought.

Referring back to the discussion of differential calculus at Detail Notes *DN 1 - Differential Calculus, Derivatives*, if

(DN5-4) $dw = f(s) \cdot ds$

then

(DN5-5) $$\frac{dw}{ds} = f(s)$$

or, in other words, $f(s)$ is the first derivative, the rate of change, of some other mathematical expression of the form *w=[some function of](s)*, which expression is the solution sought. If one can find this *[some function of]*, this $w(s)$, the anti-derivative of $f(s)$, then the problem is solved. The solution would then exist in the form of a mathematical expression which need only be evaluated between the limits of the range over which the summing up is to take place.

By "anti-derivative" is meant, of course, the expression that one had before taking the derivative. The anti-derivative is called the "integral" and anti-differentiation, the finding of the anti-derivative, is called "integration". The integration symbol is ∫ so that one can apply that operator to equation DN5-5 as

(DN5-6) $$\int \frac{dw}{ds} = \int f(s)$$

or, as it is more commonly expressed because of the way in which the problem becomes posed (as in equations DN5-3 and 4, above),

(DN5-7) $$dw = f(s) \cdot ds$$

$$w = \int dw = \int f(s) \cdot ds = \text{anti-derivative of } f(s)$$

Referring back to Figure DN5-2 and equation DN5-2, the quantity being calculated, δw, is the product of the width and the height of the approximating rectangle; it is the area of the rectangle. Thus the overall quantity sought, w, is, to an approximation, the summation of the area of all of the approximating rectangles from $s=0$ to $s=S_r$, which is an approximation to the area of the shaded triangle under the descending line that represents the force of the spring as the distance varies. When the width of each rectangle becomes infinitesimal and the number of rectangles correspondingly very large, then the summation, now an accurate valuation of w, is the area of the triangle.

The anti-derivative, $w(s)$, is an expression for that area as a function of the distance, s. If it is evaluated for a particular value of s the result is all of the area under the line over to that value of s. In the above example, if $w(s)$ were evaluated for $s=S_r$ and from that were subtracted the value for $s=0$, the result would be the area under the line of equation DN5-1 between $s=0$ and $s=S_r$, the shaded area in Figure DN5-2. That type of calculation is symbolized by placing the upper and lower limits to the upper right and lower right of the integration sign. Using that convention, the original problem's solution becomes

(DN5-8) $$w = \int_0^{S_r} f(s) \cdot ds = \int_0^{S_r} (F_0 - k \cdot s) \cdot ds$$

which leaves only the remaining problem of how to find the anti-derivative, how to perform the integration.

Unfortunately, except in simple cases there is no general procedure for integration. It is done by choosing a likely anti-derivative and checking it out by differentiating it. This trial and error process has been done in a large number of cases and the results are published as "Tables of Integrals". Sometimes, however, the function is apparently un-integrable and other methods, such as actual numerical evaluation by adding up minute rectangular areas, must be used. In that type of case one cannot obtain a mathematical expression for an answer.

One technique for integrating awkward expressions is that called "integration by parts". This procedure restates the expression as an alternative one using the relationship of equation DN1-11 at the end of Detail Notes *DN 1 - Differential Calculus, Derivatives*.

(DN1-11)
$$\frac{d(u\cdot v)}{dt} = u\cdot\frac{dv}{dt} + v\cdot\frac{du}{dt}$$

from which

(DN5-9)
$$d(u\cdot v) = u\cdot dv + v\cdot du$$

and from which

(DN5-10)
$$u\cdot dv = d(u\cdot v) - v\cdot du$$

which by integrating becomes

(DN5-11)
$$\int u\cdot dv = \int d(u\cdot v) - \int v\cdot du$$
$$= u\cdot v - \int v\cdot du$$

The example being pursued here is readily integrable, however so that, from equation DN5-8,

(DN5-8)
$$w = \int_0^{S_r}(F_0 - k\cdot s)\cdot ds = \left[F_0\cdot s - k\cdot\frac{s^2}{2}\right]_0^{S_r}$$
$$= \left[F_0\cdot S_r - k\cdot\frac{{S_r}^2}{2}\right] - \left[F_0\cdot 0 - k\cdot\frac{0^2}{2}\right]$$
$$= S_r\left[F_0 - k\cdot\frac{S_r}{2}\right]$$

is obtained. Then, from equation DN5-1, when $s=S_r$, the force is $f=0$ so that $k=F_0/S_r$.

Substituting that value in the above yields

(DN5-9)
$$w = \tfrac{1}{2}\cdot F_0\cdot S_r$$

the area of the triangle, as it should.