Appendix C

Derivation of Coulomb's Law

The fundamentals of what is known about the actions of electric charge can be summarized as follows.

- Electric charges exist in two different forms, termed the *sign* or *polarity* of the charge, "positive" or "negative".
- The charges exert a <u>force</u> on other electric charges.
- The <u>force</u> attracts or repels for charges of opposite or like signs.
- The <u>force</u> is inversely proportional to the square of the separation distance of the charges and directly proportional to the amounts of the charges.
- The effect extends throughout space.
- The charges only exist as a component effect of particles having mass which particles have been shown in the preceding sections to be *Spherical-Centers-of-Oscillation*.

The details of this behavior have been thoroughly worked out in terms of mathematics which describe the location, amount, and direction of the effect. The physical constants needed to give correct quantitative results have been well determined.

Each electrically forc**ing** particle *[Spherical-Center-of-Oscillation]* must communicate to each electrically forc**ed** particle *[Spherical-Center-of-Oscillation]* the direction from the forc**ing** particle to the forc**ed** one [for same signs repulsion], the direction from the forc**ed** particle to the forc**ing** one [for opposite signs attraction] and the magnitude and sign of the forc**ing** particle's charge. That task is assigned by contemporary physics' theory to an *electric field*, a vector field that is an assignment of a direction of action and its magnitude to each point in a region of space.

However, that designation of the field, while facilitating the description of the action fails to explain the cause, the mechanism of the field and thus fails to explain or account for the action at issue. It also fails to account for the time delay, due to the limitation of the speed of light, that must exist between a change at the forc**ing** particle and its effect at the forc**ed** particle

A flow, flowing at the speed of light, continuously, carrying the direction and magnitude information, spherically outward, from every electrically acting *Spherical-Center-of-Oscillation* to every other such *Spherical-Center-of-Oscillation*, from every charge to every other, is required. That *Propagated Outward Flow* was introduced and described in Section 3.

HOW THE CHARGES AND THEIR FLOW REPEL AND ATTRACT

The effect of an individual wave of that *Propagated Outward Flow* encountering another *Spherical-Center-of-Oscillation* is the delivery of a train of impulses to the center, Figure C-1, each an amount of momentum. That is

(C-1) impulse = force·time = mass·velocity = momentum

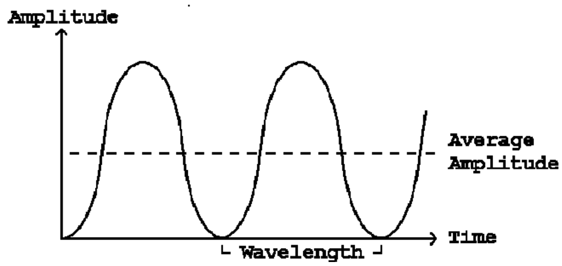

Figure C-1
The +U Wave of the Propagated Outward Flow from a +U Spherical Center-of-Oscillation

The wave as it is propagated by its source *Spherical-Center-of-Oscillation*, carries potential impulse, "potential" because it is not realized in an effect until an encounter with another *Spherical-Center-of-Oscillation* occurs. The amount of potential impulse in the wave is, of course, proportional to the amplitude of the wave. It is that amplitude, which decreases as the square of the distance from the source *Spherical-Center-of-Oscillation* because it becomes spread over a greater area.

The overall stream of waves carries the potential impulse of one wave times the repetition rate, the frequency, of the waves. The potential status of the wave's impulse is exactly the same status as that of electric field (which it, in fact, is) where electric field is potential force and not realized as actual force until it interacts with another charge.

A *Spherical-Center-of-Oscillation* propagating a *+U* Wave *Propagated Outward Flow* experiences an equal *Spherical-Center-of-Oscillation* magnitude, opposite direction reaction to the radially outgoing train of impulses as if the *Spherical-Center-of-Oscillation* were under spherical compression, Figure C-2. However, that is to no net effect because of its spherical symmetry.

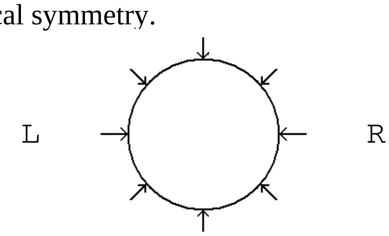

Figure C-2
The +U Spherical-Center-of-Oscillation's Reaction Back On Itself by Its Outward Flow

The train of impulses of Figure C-1 encountering the *Spherical-Center-of-Oscillation* of Figure C-2 on its left side [L] adds additional momentum to the reaction directed to the right. That being now greater than the opposing reaction to the left on the right side [R], there is now a net momentum increment to the right, a repelling action of one positive charge on another.

If the *Spherical-Center-of-Oscillation* of Figure C-2 were a *-U* center the effect would be reversed. The train of *+U* impulses of Figure C-1 encountering the center of Figure C-2 as a *-U* center on its left side [L] subtract from or cancel part of its reaction directed to the right. That being smaller than the opposing reaction to the left on the right side [R], there is a net momentum increment to the left. The effect is an attracting action of a positive charge on a negative one.

The effects and action are exactly analogous for the two other cases of a *-U Spherical-Center-of-Oscillation*'s train of *-U* impulses encountering another *-U Spherical-Center-of-Oscillation* or a *+U* one.

It is important to observe that the direction of momentum actions is the direction of the *Propagated Outward Flow* transmitting them whereas the sign or polarity *+U* or *-U* pertains back to the origin of the oscillations that started the "Big Bang" – a pair of exact opposites necessary to maintain conservation.

Having obtained from the *Spherical-Centers-of-Oscillation* and their *Propagated Outward Flows* the directions and polarities of Coulomb's Law it is now necessary to definitively quantify the action.

NEWTON'S LAW AND CENTERS & WAVES – "RESPONSIVENESS"

Newton's Second Law and as restated by inversion are:

(C-2) *Force = Mass · Acceleration*

Acceleration Resulting = Force Applied × 1/Mass

which translates in terms of the waves of *Propagated Outward Flow*s and *Spherical-Centers-of-Oscillation*s into

$$\text{(C-3)}\quad \begin{bmatrix}\text{Acceleration}\\ \text{Resulting}\end{bmatrix} = \begin{bmatrix}\text{Wave}\\ \text{Impulse}\end{bmatrix} \cdot \begin{bmatrix}\text{Responsiveness}\\ \text{of the Center}\end{bmatrix}$$

or, more succinctly,

Acceleration = Wave × Responsiveness.

Of the total wave traveling outward from the source *Spherical-Center-of-Oscillation*, the only part that interacts with another *Spherical-Center-of-Oscillation* is the part intercepted by the encountered center. The *Spherical-Center-of-Oscillation* intercepting the larger portion of incoming wave receives the greater impulse, the greater momentum change. Thus center responsiveness depends on the encountered center's cross-section target for interception of *Propagated Outward Flo*w waves.

(This analysis assumes that the part of the wave intercepted by the encountered center is a flat wave front. The non-plane wave case, for small separation distances, is in most cases of negligible effect except the slight "Lamb Shift" treated in Appendix A-1, *The Neutron*, Likewise, because δ, the encountered particle's core radius, is so minute the target can be deemed flat)

A *Spherical-Center-of-Oscillation* of smaller cross-section is of greater mass (lesser responsiveness). The encountered center being a spherical oscillation the cross-section is the area of a circle perpendicular to the direction of travel of the incoming wave front as it encounters the center. That area is proportional to π times the square of the center's wavelength .

This yields the first factor in *Spherical-Center-of-Oscillation* responsiveness,

(C-4) $$\text{Cross-section} \propto \pi \cdot \lambda_c^2 = K_{cs} \cdot \lambda_c^2$$

(C-5) $$\begin{bmatrix}\text{respon-}\\ \text{siveness}\end{bmatrix} \propto [\text{Factor 1}] \cdot [\text{Factor 2}] \cdot [\text{Factor 3}]$$
$$= [K_{cs} \cdot \lambda_c^2] \cdot [\quad " \quad] \cdot [\quad " \quad]$$

where: K_{cs} = a constant for the proportionality
λ_c = the encountered center oscillation wavelength

The incoming wave must be expressed in terms of "Wave Impulse per Unit Area" so that multiplied by the cross-section area at the encountered *Spherical-Center-of-Oscillation* the units of area are cancelled and the resulting quantity is wave impulse.

(C-6) $$\text{"Wave"} = \frac{\text{Total Source Center Propagated Wave}}{\text{Total Spherical Area of Source Wave at Distance Encountered Center is from Source}}$$
$$= \text{Wave Impulse per Unit Area}$$

Factor 2 in the responsiveness, Equation *C-5* is the effective amplitude of the *Spherical-Center-of-Oscillation*s's oscillation. A range of possible interactions can occur because the source and encountered center frequencies may differ. The extremes and mean of the range of encounters follow.

(1) $\text{Frequency}_{source} << \text{Frequency}_{encountered}$

The encountered center goes through all of its amplitude values many times while one source wave arrives. Its effective amplitude is its average amplitude.

(2) $\text{Frequency}_{source} >> \text{Frequency}_{encountered}$

The source center goes through all of its amplitude values many times while the encountered does once. Its effective amplitude is its average amplitude.

(3) $\text{Frequency}_{source} = \text{Frequency}_{encountered}$

The interaction takes place over exactly one cycle and the effective amplitude is, again, the average.

In real matter, not the idealized model of one source and one encountered center, every *Spherical-Center-of-Oscillation* is constantly "bombarded" by various waves from a variety of directions at a variety of frequencies and phases due to the immense number of *Spherical-Centers-of-Oscillation* making up ordinary matter. The relative frequency and the phase of the wave and the encountered center have no effect on the large scale result from the interaction. Thus *Factor 2* is not a variable quantity but merely the average amplitude of the encountered center, which is designated U_C.

However, the absolute frequency of the encountered *Spherical-Center-of-Oscillation* is *Factor 3* in the formula for responsiveness. Just as the incoming wave repetition rate affects the amount of force that the wave can deliver to the encountered center, so the encountered center repetition rate affects that center's response to the wave. While the wave is encountering the center, each cycle of the encountered center's oscillation is acted on by the wave. (This is most easily visualized if the frequency of the encountered center is much larger than that of the wave, but it applies in any case.)

Thus *Factor 3* is encountered center repetition rate. [For a center at rest the "rep rate" is the oscillation frequency but for a center in motion its velocity is a factor in the "rep rate" along with its oscillation frequency.

Then Equation *C-5* becomes

(C-7) $$\text{responsiveness} \propto [\text{cross-section}]\cdot[\text{amplitude}]\cdot[\text{rep rate}]$$
$$= [K_{cs}\cdot\lambda_c^2]\cdot[U_c]\cdot[f_c]$$
$$= K_{cs}\cdot\lambda_c\cdot U_c\cdot c \qquad [\text{Using } c = f\cdot\lambda]$$

where: K_{cs} = a constant for the proportionality
λ_c = the encountered center oscillation wavelength
U_c = its amplitude, and
f_c = its frequency.

PRECISE FORMULATION OF COULOMB'S LAW

The treatment here is of one single unit charge, $\pm U_c\cdot[1 - \cos(2\pi\cdot f\cdot t)]$, interacting with another such single unit charge, one simple basic *Spherical-Center-of-Oscillation* interacting with another.

The Encountered Center Charge Q_e and Its Amplitude U_c

In the traditional formulation of Newton's Law Equation *C-8*

(C-8) $$\text{Force} = \text{mass}\cdot\text{acceleration}$$

and for the case that is now being considered, that in which the force results from the electrostatic interaction between two charges in accordance with Coulomb's Law, Equation *C-9*,

(C-9) $$\text{Force} = \frac{\text{Charge}\cdot\text{Charge}}{\text{Separation Distance}^2}$$

both of the charges enter into the relationship in the *Force* part, the *Mass* part of the relationship being like an inert characteristic of the substance.

In this *Centers-of-Oscillation* formulation Equation *C-2*, repeated here

(C-2) $$\begin{bmatrix}\text{Acceleration}\\ \text{Resulting}\end{bmatrix} = \begin{bmatrix}\text{Wave}\\ \text{Impulse}\end{bmatrix}\cdot\begin{bmatrix}\text{Responsiveness}\\ \text{of the Center}\end{bmatrix}$$

or, more succinctly,

$$\text{Acceleration} = \text{Wave} \times \text{Responsiveness}$$

the amplitude of the oscillation, U_c for the center, U_w for the wave, the role of which corresponds to that of traditional charge, Q, enters into the formulation differently from the traditional conception. The source *Spherical-Center-of-Oscillation*'s amplitude is a factor in the Wave and the encountered *Spherical-Center-of-Oscillation*'s amplitude is a factor in the Responsiveness.

Figure C-3 on the following page compares the two.

Field and Wave, not Force and Wave, correspond. Each is the unrealized potential that becomes action via interaction with an encountered charge / center. Therefore the *[Charge ÷ Mass]* of the left half of Figure C-3 is the same as the *Responsiveness* of the right half of the figure.

Traditional — *Centers - of - Oscillation*

$$\text{Acceleration} = \text{Force} \times \left[\frac{1}{\text{Mass}}\right] \qquad \text{Acceleration} = \left[\begin{matrix}\text{Wave}\\ \text{Impulse}\end{matrix}\right] \times \text{Responseness}$$

$$= \left[\frac{Q\cdot Q}{d^2}\right] \times \left[\frac{1}{\text{Mass}}\right]$$

$$= \left[\frac{Q_s}{d^2}\right] \times \left[\frac{Q_e}{\text{Mass}}\right]$$

$$= \left[\begin{matrix}\text{Electric}\\ \text{Field at } d^2\end{matrix}\right] \times \left[\frac{Q_e}{\text{Mass}}\right] \qquad = \left[\begin{matrix}\text{Wave}\\ \text{Impulse}\end{matrix}\right] \times \left[K_{cs}\cdot\lambda_c\cdot U_c\cdot c\right]$$

Figure C-3

Therefore

(C-10) $$\frac{Q_e}{m_e} = K_{cs}\cdot\lambda_c\cdot U_c\cdot c$$

from which

(C-11) $$Q_e = \frac{h}{\lambda_c\cdot c}\cdot\left[K_{cs}\cdot\lambda_c\cdot U_c\cdot c\right] \qquad [\text{Using } m\cdot c^2 = h\cdot f]$$
$$= h\cdot K_{cs}\cdot U_c$$

which relates the charge of the encountered *Spherical-Center-of-Oscillation* to it's amplitude, and is a simple direct proportionality because h and K_{cs} are constants.

The Source Center Charge Q_s and Its Oscillation Amplitude U_c

If time could be stopped so that the waves from the source center were frozen in whatever position that they had in space, then the spherical waves as propagated by a *Spherical-Center-of-Oscillation* would appear as a series of nested shells, each of a successively greater radius, R, the radius being

(C-12) $$R_w = n\cdot\lambda_w$$

where: $n = 1, 2, 3 \ldots$ for the successive shells
λ_w = the wavelength of the waves

and the thickness of each shell is the wavelength, λ_w. One such shell is depicted two-dimensionally in Figure C-4, below.

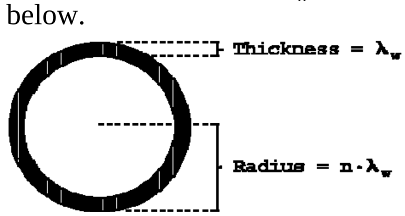

Figure C-4

A cross-sectional view of this wave in space, that is a graph of its amplitude variation along a radius while traversing the thickness, is depicted in Figure C-5, below, where it is clear that the area under the curve of amplitude variation is equal to $U_w \cdot \lambda_w$.

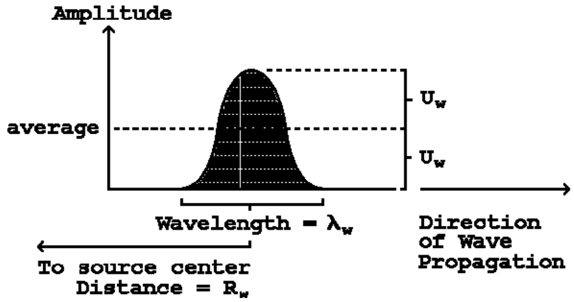

Figure C-5

The potential impulse in one complete spherical shell, one wave cycle, is the shell cross-section, $U_w \cdot \lambda_w$, multiplied by the spherical surface area of the shell, $4\pi \cdot R_w$.

(C-13) $[\text{a cycle of wave impulse}] = [U_w \cdot \lambda_w] \cdot [4\pi \cdot R_w]$

But, the wave amplitude, U_w, is the *Spherical-Center-of-Oscillation*'s amplitude, U_c, divided by the area of the wave's spherical shell at R_w and $\lambda_w = \lambda_c$ so that

(C-14) $[\text{a cycle of wave impulse}] = U_c \cdot \lambda_c$

The *Wave* of Figure C-3 is the Equation *C-14* single *[a cycle of wave impulse]* multiplied by the repetition rate, the frequency, $f_w = f_c$, so that the wave, of Figure C-3 is

(C-15) $\text{Wave} = [U_c \cdot \lambda_c] \cdot f_c = U_c \cdot c = Q_s$,

which relates the field of the source *Spherical-Center-of-Oscillation* to that center's oscillation amplitude and, therefore, relates the charge of the source center to its amplitude.

Recognizing that every *Spherical-Center-of-Oscillation* is always in both source and encountered roles, then setting Equation *C-11* equal to Equation *C-14* the following is obtained.

(C-16) $Q_e = Q_s$

$h \cdot K_{cs} \cdot U_c = U_c \cdot c$

therefore

$Q = U \cdot c \quad \text{and} \quad K_{cs} = {}^{c}/_{h}$

Two Such Charges Interact Electrostatically As Follows

(1) The total potential force in the wave series as propagated by the source *Spherical-Center-of-Oscillation*s is (from Equation *C-15*)

(C-15) $U_c \cdot c$

(2) The total wave series potential force per unit area of wave front at the encountered *Spherical-Center-of-Oscillation* is the quantity of step (1) divided by the spherical surface at the encountered center.

(C-17) $$\frac{U_c \cdot c}{4\pi \cdot R^2}$$

(3) The responsiveness of the encountered *Spherical-Center-of-Oscillation* is (Equation *C-7*)

(C-7) $$\text{Responsiveness} = K_{cs} \cdot \lambda_c \cdot U_c \cdot c$$

(4) The resulting acceleration is, therefore (substituting steps (2) and (3), above, into Equation *C-3* per Equation *C-6*)

(C-18) $$\text{Acceleration} = \begin{bmatrix} \text{Wave Potential} \\ \text{Impulse per Unit} \\ \text{Area at the En-} \\ \text{countered Center} \end{bmatrix} \cdot \begin{bmatrix} \text{Responsiveness} \\ \text{of the} \\ \text{Encountered} \\ \text{Center} \end{bmatrix}$$

$$= \frac{U_c \cdot c}{4\pi \cdot R^2} \cdot K_{cs} \cdot \lambda_c \cdot U_c \cdot c$$

(5) The mass of the encountered *Spherical-Center-of-Oscillation* (from $m \cdot c^2 = h \cdot f$) is

(C-19) $$m = \frac{h}{c \cdot \lambda_c}$$

(6) The force is, then (substituting steps (4) and (5), above into Equation *C-2*)

(C-20) $$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$= \left[\frac{h}{c \cdot \lambda_c}\right] \cdot \left[\frac{U_c \cdot c}{4\pi \cdot R^2}\right] \cdot K_{cs} \cdot \lambda_c \cdot U_c \cdot c$$

$$= \frac{[U_c \cdot c] \cdot [h \cdot K_{cs} \cdot U_c \cdot c]}{4\pi \cdot R^2}$$

and substituting per Equations *C-11* and *C-15* yields the result

(C-21) $$\text{Force} = \frac{Q_s \cdot Q_e}{4\pi \cdot R^2}$$

which is Coulomb's law as it naturally occurs.

If a constant of proportionality, k, is introduced to accommodate choice of the units of charge, and the 4π is absorbed into that new constant, then the result (using q for charge since the added constant requires an accordingly different variable) is

(C-22) $$\text{Force} = k \cdot \frac{q_s \cdot q_e}{R^2} \qquad k = {}^{1}/_{4\pi\varepsilon_0}$$

which is Coulomb's Law as originally formulated.

[* See on the following page the analysis

"Understanding:

The Units of Charge and of Coulomb's Law"]

Understanding:

The Units of Charge and of Coulomb's Law

Properly stated, the law of electrostatic interaction between two charges, called Coulomb's Law, is

> "Given two electric charges separated in space by some distance, the magnitude of the force exerted by each of the charges on the other is directly proportional to the product of the charges and inversely proportional to the square of the distance between them."

In symbols this is

(C-A-1)

$$F = k \cdot \frac{Q_1 \cdot Q_2}{R^2}$$

where k = the constant of the proportionality.

Unfortunately the manner in which the law was originally formulated and other complications led to various systems of units.

It is desirable for simplicity that the units for the quantities in such laws be so as to have the constant of proportionality, k, be unity. Then the constant of proportionality can be omitted and the statement of the law involves only the actual variables pertinent to the law.

There are many examples of physical laws in which this was accomplished:

Force = Mass × Acceleration
(not k × Mass × Acceleration)

Voltage = Current × Resistance
(not k × Current × Resistance)

and so forth.

Of course, what is desired is that this be done ($k=1$) successfully for all systems of units that might be used. Commonly encountered systems of units are:

(1) cgs ≡ length in centimeters (cm)
mass in grams (gm)
time in seconds (sec)

with other units following accordingly as prescribed by the physical laws involved, for example:

$$\begin{aligned} \text{force} &= \text{mass} \times \text{acceleration} \\ &= \text{mass} \times {}^{\text{length}} / \text{time}^2 \\ &= \text{gm} \times {}^{\text{cm}}/\text{sec}^2 \\ &\equiv \text{dyne} \end{aligned}$$

(2) MKS ≡ length in meters (m)
mass in kilograms (kg)
time in seconds (sec)

$$\begin{aligned} \text{force} &= \text{kg} \times {}^{\text{m}}/\text{sec}^2 \\ &\equiv \text{newton} \end{aligned}$$

It would appear then that one need merely rearrange Coulomb's Law so that it can be used to define the units of charge as

(C-A-2) $$Q^2 = \frac{F \times R^2}{k}$$ [C-A-1 rearranged, and since this is only for units, $Q_1=Q_2=Q$]

and an orderly, simple arrangement of units would result. But, unfortunately it does not. From equation *C-A-2* the units of *Q* are as follows.

(C-A-3) $$Q = [F \times R^2]^{1/2}$$ [k is dimensionless for finding the natural units of Q]

$$= [\text{force} \times \text{length}^2]^{1/2}$$

$$= [(\text{mass} \times \text{acceleration}) \times \text{length}^2]^{1/2}$$

$$= \left[\text{mass} \times \frac{\text{length}}{\text{time}^2} \times \text{length}^2\right]^{1/2}$$

$$= \left[\frac{\text{mass} \times \text{length}^3}{\text{time}^2}\right]^{1/2}$$

The following table, Figure C-A-1, indicates the manner in which common units work out in this formulation in different systems of units.

Quantity	cgs Units	MKS Units	Ratio cgs/MKS
length	cm	m	10^2 cm/m
mass	gm	kg	10^3 gm/kg
time	sec	sec	1
velocity	cm/sec	m/sec	10^2 cm/m per sec
force	dyne	newton	10^5 dyne/newton
. . . .	. . . .	. . . .	. . . .
charge	$\left[\frac{\text{gm} \times \text{cm}^3}{\text{sec}^2}\right]^{1/2}$	$\left[\frac{\text{kg} \times \text{m}^3}{\text{sec}^2}\right]^{1/2}$	$10^{9/2} = 10^4 \cdot \sqrt{10}$ cgs units per MKS unit

Figure C-A-1

Most quantities in the table experience value changes by a multiple factor of ten in going from *cgs* to *MKS* units, which is simple. The digits of a quantity remain the same, only the position of the decimal point changes from one system of units to another. But charge will not fit that simple pattern. If for example a certain value of charge in some situation in *cgs* units were to be $4.803 \cdot 10^{-10}$ the corresponding value in *MKS* units would be $1.519 \cdot 10^{-14}$. That is not simple and orderly as desired because the digits of the quantity as well as the power of ten change with the change in units used.

A second complication arises when it is further found that the k in Coulomb's law is not always the same. Its value depends on the nature of the material substance (or lack of it) intervening between the two charges, for example: air, glass, oil, free space (perfect vacuum), etc.

Recognizing, then, that the desirable procedure of choosing the fundamental unit of new quantities so that $k = 1$ is hopeless in this case of charge, k is set up as a constant that is dependent on the intervening material and retained as part of the physical law. The k is designated $1/\varepsilon$ (Greek letter epsilon). For free space the epsilon is designated as ε_0. For one system of units the "natural electrostatic units" (and for the free space condition) $k = 1$ can still be retained as $\varepsilon_0 = 1$.

Coulomb's Law then becomes

(C-A-4) $$F = \frac{Q_1 \cdot Q_2}{\varepsilon \cdot R^2} \qquad \text{[Anywhere]}$$

and

(C-A-5) $$F = \frac{Q_1 \cdot Q_2}{\varepsilon_0 \cdot R^2} \qquad \text{[In free space].}$$

The more or less orderly arrangement in Figure C-A-2, below, results as established in practice, and is as simple and orderly as can be obtained in the circumstances. The "Elemental Charge" in the table is the value of the charge of an electron or a proton, the fundamental electric charge of the universe, in each system of units.

The esu (cgs, natural electrostatic) units is the system in which Coulomb's Law was originally developed (implicitly), where the value of the charge is $4.803 \cdot 10^{-10}$ and $\varepsilon_0 = 1$. The units, abcoulombs, are equal to statcoulombs divided by the velocity of light, c. A *coulomb* is *10 abcoulombs*. The rationalized system of units recognizes the significance of the 4π factor in the law and takes it into the ε_0 rather than the more awkward step of changing the charge to its otherwise value times $1/\sqrt{4\pi}$.

(For a thorough analysis of systems of units see Chapter 3, *Handbook of Engineering Fundamentals*, First Edition, Ovid W. Eshbach, New York, John Riley & Sons, 1947.)

The *Rationalized Meter - Kilogram - Second (MKSR*) system in Figure C-A-2, below, is now established as the standard system of units to be used internationally. The system is now referred to as *SI Units*, that is *Standard International* Units.

System of Units	Elemental Charge	Value of ε_0	Correct Statement of Coulomb's Law For in Free Space
esu (cgs, natural electrostatic)	$4.803 \cdot 10^{-10}$ statcoulombs (Q)	1	$F = \frac{Q_1 \cdot Q_2}{\varepsilon_0 \cdot R^2} = \left[\frac{Q_1 \cdot Q_2}{R^2}\right]$
emu (cgs, natural electromagnetic with ε as it "naturally" occurs, see after the Figure)	$1.602 \cdot 10^{-20}$ abcoulombs (q_{abs})	$\frac{1}{c^2}$	$F = \frac{q_{1abs} \cdot q_{2abs}}{\varepsilon_0 \cdot R^2} = \frac{c^2 \cdot q_{1abs} \cdot q_{2abs}}{R^2} = \left[\frac{Q_1 \cdot Q_2}{R^2}\right]$
MKS (MKS with an adjustment factor in ε)	$1.602 \cdot 10^{-19}$ coulombs (q)	$\frac{10^7}{c^2}$	$F = \frac{q_1 \cdot q_2}{\varepsilon_0 \cdot R^2} = \frac{c^2 \cdot q_1 \cdot q_2}{10^7 \cdot R^2}$
		* → ** →	$= \frac{(10^4 \cdot \sqrt{10})^2}{10^2} \cdot \frac{c^2 \cdot q_{1abs} \cdot q_{2abs}}{10^7 \cdot R^2} = \left[\frac{Q_1 \cdot Q_2}{R^2}\right]$
MKSR ("rationalized" MKS)	$1.602 \cdot 10^{-19}$ coulombs (q)	$\frac{10^7}{4\pi \cdot c^2}$	$F = \frac{q_1 \cdot q_2}{4\pi \cdot \varepsilon_0 \cdot R^2} = \frac{c^2 \cdot q_1 \cdot q_2}{10^7 \cdot R^2} = \left[\frac{Q_1 \cdot Q_2}{R^2}\right]$

* (This is the cgs/MKS units ratio applied to each charge.)

** (The abcoulomb/coulomb ratio applied to each charge.)

Figure C-A-2