SECTION 6

The Action of Matter: Matter Waves

The Matter Wave Problem

In the early 20th Century it was proposed, in the absence of any knowledge of *Spherical-Centers-of-Oscillation*, that perhaps matter, which was accepted as being particle in nature might sometimes exhibit wave behavior. It was hypothesized that the wave aspect of a particle of matter should have a wavelength, λ_{mw}, of

(6-1)
$$\lambda_{mw} = \frac{h}{\text{particle momentum}} = \frac{h}{m \cdot v}$$

This was soon verified by the obtaining of electron diffraction patterns whose observed wavelengths corresponded well enough with the prediction. At that point one would think that the duality of matter was enough established that extensive further investigation of matter waves would have resulted. But that was not the case and the reason was a fundamental problem that could not be overcome – the matter wave frequency.

If one reasons that the kinetic energy of the particle of matter should correspond to its matter wave frequency, f_{mw}, as

(6-2)
$$f_{mw} = \frac{W_k}{h} = \frac{\frac{1}{2} \cdot m \cdot v^2}{h}$$

then the velocity of the matter wave is

(6-3)
$$v_{mw} = \lambda_{mw} \cdot f_{mw} = \left[\frac{h}{m \cdot v}\right] \cdot \left[\frac{\frac{1}{2} \cdot m \cdot v^2}{h}\right] = \frac{1}{2} \cdot v$$

which states that the matter wave moves at one half the speed of the particle. That is obviously absurd as they must move together each being merely an alternative aspect of the same real entity.

It is no help in resolving this difficulty if relativistic mass is used (as it should be in any case) since the same mass appears in both numerator and denominator of equation *6-3* where they simply cancel out.

It is also no help to hypothesize that it is the total energy, not just the kinetic energy, that yields the matter wave. Such an attempt attributes a matter wave to a particle at rest. It also gives the resulting matter wave velocity as c^2/v which has the matter wave racing ahead of its particle.

It was the inability to resolve this problem that led to the loss of interest in matter waves and essentially the end of further inquiry with regard to the wave aspect of matter.

Resolution of the Matter Wave Problem

If instead of kinetic energy one uses energy in kinetic form, $m_v \cdot v^2$, as developed in Section 4, *The Action of Matter: Motion and Relativity*, equation *4-13* the problem of the matter wave frequency is resolved. The traditional view of kinetic energy as the energy increase due to motion is not valid as a description of the processes taking place.

Using mass- and energy-in-kinetic-form to obtain the frequency of the matter wave proceeds as follows.

(6-4) $$f_{mw} = \frac{m_v \cdot v^2}{h}$$ [equation *6-2*, but using W_v, equation *4-13*, energy-in-kinetic-form, for W_k, kinetic energy]

Using this result for matter wave frequency and using the same relativistic mass, m_v, in equation *6-5* for the matter wavelength the velocity of the matter wave then is

(6-5) $$v_{mw} = f_{mw} \cdot \lambda_{mw} = \left[\frac{m_v \cdot v^2}{h}\right] \cdot \left[\frac{h}{m_v \cdot v}\right] = v$$

and the wave is traveling with and as the particle. On that basis the wave aspect of matter is established both experimentally and theoretically.

Matter Waves and Spherical Centers of Oscillation

The matter wave traveling right along with the particle is like a kind of standing wave relative to the particle. A standing wave can be thought of as the sum result of two waves traveling in opposite directions through each other. If the frequencies and wavelengths are different then their interaction produces a new frequency called a "beat". The development of the beat is as follows.

The two waves are

(6-6) $$\text{Wave \#1} = A \cdot \text{Sin}(2\pi f_1 t)$$

$$\text{Wave \#2} = A \cdot \text{Sin}(2\pi f_2 t)$$

and the sum is

(6-7) $$\text{WaveSum} = A \cdot \text{Sin}\left[2\pi f_1 t\right] + A \cdot \text{Sin}\left[2\pi f_2 t\right]$$

which by using a trigonometric equivalence can be arranged as

$$\text{WaveSum} = 2A \cdot \text{Sin}\left[2\pi \frac{f_1 + f_2}{2} t\right] \cdot \text{Cos}\left[2\pi \frac{f_1 - f_2}{2}\right]$$

The cosine term frequency *½·[f1-f2]* difference, is smaller than the sine term sum *½·[f1+f2]*. If the expression is viewed as the higher frequency sine portion with the rest of the expression being the amplitude, as in equation *6-8*, then

(6-8) $$\text{WaveSum} = \left[2A \cdot \text{Cos}\left[2\pi \frac{f_1 - f_2}{2} t\right]\right] \cdot \text{Sin}\left[2\pi \frac{f_1 + f_2}{2} t\right]$$

$$= \left[\text{Varying Amplitude}\right] \cdot \text{Sin}\left[2\pi \frac{f_1 + f_2}{2} t\right]$$

The wave form appears as in Figure 6-1, below.

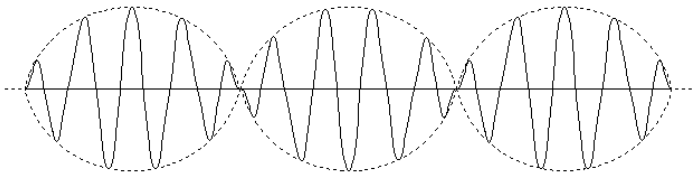

Figure 6-1

The solid-line curve in Figure 6-1 is the overall wave form. The dotted line, the *envelope*, is the varying amplitude. The overall wave form exhibits in the varying amplitude a periodic variation called the *beat*. The beat is real, not merely an appearance. For example two sound tones heard simultaneously produce an audible beat that one can hear. It is by listening to the beat that one tunes a piano or other musical instrument.

Matter waves are the beat that results from the *Spherical-Center-of-Oscillation*'s forward and rearward oscillations interacting with each other. This develops as follows. For a center in motion at velocity v, per Figure 4-3

(6-9) $\lambda_{fwd} = \lambda_v \cdot (1 - v/c)$ $\quad f_{fwd} = c/\lambda_{fwd}$

$\lambda_{rwd} = \lambda_v \cdot (1 + v/c)$ $\quad f_{rwd} = c/\lambda_{rwd}$

The beat frequency, using the "Varying Amplitude" portion of equation 6-8, substituting f_{fwd} for f_1 and f_{rwd} for f_2, and then using equation 6-9, is

(6-10)
$$f_{beat} = \frac{1}{2}[f_{fwd} - f_{rwd}] = \frac{1}{2}\left[\frac{c}{\lambda_v\left[1-\frac{v}{c}\right]} - \frac{c}{\lambda_v\left[1+\frac{v}{c}\right]}\right]$$

$$= \frac{c}{2\cdot\lambda_v}\cdot\left[\frac{\left[1+\frac{v}{c}\right]-\left[1-\frac{v}{c}\right]}{\left[1-\frac{v}{c}\right]^2}\right] = \frac{v}{\lambda_v}\cdot\left[\frac{1}{\left[1-\frac{v}{c}\right]^2}\right]$$

$$\lambda_{beat} = \frac{c}{f_{beat}} = \lambda_v \cdot \frac{c}{v}\cdot\left[1-\frac{v^2}{c^2}\right]$$

Substitute Eqn 4 – 2: $\lambda_v = \lambda_r \cdot \frac{1}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}$

$$= \left[\lambda_r \frac{1}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}\right]\cdot\frac{c}{v}\cdot\left[1-\frac{v^2}{c^2}\right]$$

Substitute per: $m\cdot c^2 = h\cdot f = h\cdot\frac{c}{\lambda} \rightarrow \lambda_r = \frac{h}{m_r\cdot c}$

$$= \left[\frac{h}{m_r\cdot c}\right]\cdot\frac{c}{v}\cdot\left[\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}\right]$$

Substitute per Eqn 4 – 6: $m_v = m_r \cdot \frac{1}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}$

$$= \frac{h}{m_v\cdot v}$$

which is the matter wavelength as previously obtained per equation 6-1 (in which the mass must be relativistic mass, m_v, of course). Thus matter waves are the beat that results from the *Spherical-Center-of-Oscillation*'s forward and rearward oscillations interacting with each other.

A moving center-of-oscillation as "seen" by an external observer appears as the waves propagated by the center in his direction appear. But, if one could, somehow, actually "see" the center itself pulsating as it does, the situation would be different. The interaction of the forward and rearward oscillations, which produce a beat at the matter wave frequency, are real. The effect is as follows (repeating the form of equations *6-6* through *6-8*, which were for any general oscillation, but now using the oscillations of a center-of-oscillation in motion).

(6-11) $$\text{Forward Wave} = A\cdot\left[1+\text{Sin}(2\pi f_1 t)\right]$$

$$\text{Rearward Wave} = A\cdot\left[1+\text{Sin}(2\pi f_2 t)\right]$$

```
[Note: 1 - cos(x) ≡ 1 + cos(180° - x)
                  ≡ 1 + sin[90°-(180° - x)]
                  ≡ 1 + sin(x - 90°)
and the 90° phase is irrelevant, of course.]
```

and the sum is

(6-12) $$\text{Wave Sum} = A\cdot\left[2+\text{Sin}\left[2\pi f_1 t\right]+\text{Sin}\left[2\pi f_2 t\right]\right]$$

Which again by using a trigonometric equivalence can be arranged as

$$\text{Wave Sum} = 2A + 2A\cdot\text{Sin}\left[2\pi\frac{f_1+f_2}{2}t\right]\cdot\text{Cos}\left[2\pi\frac{f_1-f_2}{2}\right]$$

The cosine term is at a lesser frequency than the sine term. If the expression for the wave sum is viewed as the (higher frequency) sine portion with the rest of the expression being the amplitude, as in equation *6-13*, then

(6-13)
$$\text{Wave Sum} = 2A\cdot\left[1+\text{Cos}\left[2\pi\frac{f_1-f_2}{2}t\right]\right]\cdot\text{Sin}\left[2\pi\frac{f_1+f_2}{2}t\right]$$

$$= 2A\cdot\left[\begin{matrix}1+\text{cosine form of}\\ \text{Varying Amplitude}\end{matrix}\right]\cdot\text{Sin}\left[2\pi\frac{f_1+f_2}{2}t\right]$$

In the case of a *Spherical-Center-of-Oscillation* $f_1 = f_{fwd}$ and $f_2 = f_{rwd}$. Likewise, A is U_C, the center average amplitude, the oscillation being of the form $U_C\cdot[1 - Cos(2\pi\cdot f\cdot t)]$ as before, equation *1-16*.

The wave form appears as in Figure 6-2, below, for the forward-rearward interaction and the matter wave beat of the center's pulsation as it would be "seen" from the side relative to its direction of motion.

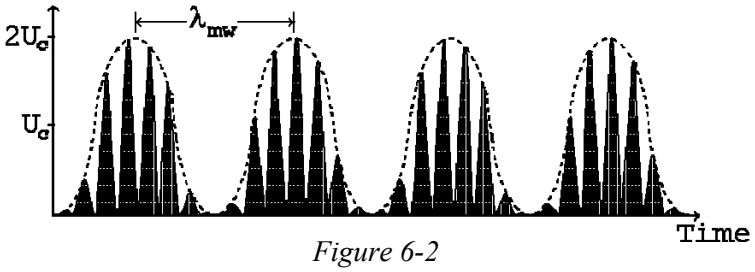

Figure 6-2
The Forward-Rearward Pulsation of a Center in Motion
Which is the Matter Wave

Matter Waves and Electron Orbits In Atoms

By the early 20th Century it had become clear that atoms consist of a positively charged minute nucleus surrounded mostly by empty space except for a moderate number of negative, equally charged, minute electrons in orbits around the nucleus the orbital configuration maintained by a balance of Coulomb attraction and centrifugal force. It also had become clear that there were only a small number of discrete orbits that were stable, that is orbits that supported a continuous cyclical electron path around the nucleus.

Intriguingly it had also been found that the stable orbits are only those whose orbital path length is exactly an integer multiple of the orbiting electron's matter wave length. One would have expected that such a significant correlation would have led to extensive further investigation of matter waves and of the correlation. However, the then unsolved problem of the matter wave frequency [first page of this section] resulted in general neglect of matter waves.

It also resulted in the invention of an alternative so far as electron orbits are concerned. The statement that the orbital electron stable path length is an integer multiple of the electron's matter wave length, equation *6-14*,

(6-14) Orbital Path = Matter Wave Length Integer Multiple

$$2\pi \cdot R = n \cdot \frac{h}{m \cdot v} = n \cdot \lambda_{mw} \qquad n = 1, 2, \ldots$$

was algebraically modified [by switching the location of the 2π and the $m \cdot v$] to state that the orbiting electron's angular momentum occurred in only integer multiples of a fundamental quantity [Planck Constant over 2π], i.e. are "quantized", equation 6-15.

(6-15) Orbital Angular Momentum = "Quantized"

$$m \cdot v \cdot R = n \cdot \frac{h}{2\pi} \qquad n = 1, 2, \ldots$$

There is no cause, no mechanism that requires the orbital angular momentum to be "quantized". But that the stable orbits are only those whose orbital path length is exactly an integer multiple of the orbiting electron's matter wave length is due to a specific behavior of the Spherical-Centers-of-Oscillation *as follows.*

Taking the simple case of the Hydrogen atom with its single proton nucleus and single orbital electron the Coulomb attraction by the positive atomic nucleus on the negative orbiting electron is not a smooth continuous action. Rather it is the result of an on-going stream of pulses at the rate of the frequency of oscillation of the proton *Spherical-Center-of-Oscillation* that is per equation *2-6b* a frequency of $2.268,731,818 \cdot 10^{23}$ hz when the proton is at rest.

For the orbit to be stable it must be the same for each pass, pass after pass. If each pass includes exactly an integer number of the orbital electron's matter wave lengths then each pass has exactly the same set of Coulomb force pulses acting in each orbital pass. But if, for example, the orbital path length contains only $^{9}/_{10}$ of a matter wave length, $^{9}/_{10}$ of the matter wave period, then the next pass will contain the missing $^{1}/_{10}$ of the matter wave length or wave period plus $^{8}/_{10}$ of the next, and so on. The matter wave being sinusoidal in form, the successive orbital passes will be all different, in Figure 6-2.

It is this behavior which operatively causes the "stable orbits", and only those orbits, to be stable. It has nothing to do with angular momentum nor quantization of angular momentum.

How Electrons Are Forced Into Stable Orbits

With the vast amount of *Propagated Outward Flow* from myriad *Spherical-Centers-of-Oscillation* orbital electrons are continuously buffeted. How are specific stable orbit paths enforced? To analyze and quantify the deviations in the variable quantities involved, the radius, R, and the electron orbital velocity, v, will be expressed in terms of the orbit number, n, the number of matter wavelengths in the orbital path. That requires obtaining expressions for them that do not include any other variables.

That quantity, n, will here be deemed to be a continuous variable so that the R and v expressed in terms of n can be continuously variable and able to address locations between stable orbits, not merely the discrete amounts at the stable orbits.

The balance of forces for stability in a circular orbit requires

(6-16) $$\text{Centrifugal Force} = \text{Centripetal Force}$$

$$\frac{m \cdot v^2}{R} = \frac{q^2}{4\pi \cdot \varepsilon \cdot R^2}$$

$$R = \frac{q^2}{4\pi \cdot \varepsilon \cdot m \cdot v^2}$$

(6-17) $$\text{Orbit Path Length} = n \cdot \text{Matter Wavelength}$$

$$2\pi \cdot R = n \cdot \frac{h}{m \cdot v}$$

$$2\pi \left[\frac{q^2}{4\pi \cdot \varepsilon \cdot m \cdot v^2} \right] = n \cdot \frac{h}{m \cdot v} \qquad \text{[Substitute R]}$$

$$v = \frac{q^2}{2\pi \cdot \varepsilon \cdot n \cdot h} \qquad \text{[Solve for v]}$$

$$v \propto \frac{1}{n}$$

(6-18) $$R = \frac{q^2}{4\pi \cdot \varepsilon \cdot m \cdot v^2} \propto \frac{q^2}{4\pi \cdot \varepsilon \cdot m \cdot \left[\frac{1}{n} \right]^2} \qquad \text{[Substitute 6-17]}$$

$$R \propto n^2$$

In those terms the variation of the required centripetal force for a circular orbit as n varies is

(6-19) $$F_{\text{Centripetal}} = \frac{m \cdot v^2}{R} \propto \frac{\left[1/n \right]^2}{n^2} = \frac{1}{n^4}$$

With constant charge the only variable in the expression for the Coulomb force is R in the denominator and is proportional to n^4. Therefore

(6-20) $F_{Coulomb} \propto {}^{1}/_{n^4}$

Thus the normal Coulomb force always provides the exact value of $F_{centripetal}$ required for a stable circular orbit.

The numerator of the Coulomb force expression is q^2. The variation from the force it exerts in the stable orbits depends on the ratio of the orbital path length, $2\pi \cdot R$, to the matter wavelength, ${}^{h}/_{m \cdot v}$. If that ratio is an integer then the behavior is the normal stable orbit Coulomb force.

If that ratio is not an integer then the force is *quasi-stable Coulomb*, as if the effective charge were modified as follows.

(6-20)
$$\text{Coulomb Force Numerator} \propto \frac{\text{Orbit Length}}{\lambda_{mw}}$$
$$\propto \frac{2\pi \cdot \mathbf{R}}{{}^{h}/_{m\mathbf{v}}} = \frac{2\pi \cdot \mathbf{R} \cdot m \cdot \mathbf{v}}{h}$$
$$\propto n^2 \cdot [{}^{1}/_{n}] = n$$
$$\text{Coulomb Force Denominator} \propto \mathbf{R}^2 \propto n^4$$

and the overall *quasi-stable Coulomb* force then varies as

(6-21)
$$F_{Quasi\text{-}Coulomb} = \frac{\text{Numerator}}{\text{Denominator}} \propto \frac{n}{n^4} = {}^{1}/_{n^3}$$

The ratio of the quasi-Coulomb force to the normal Coulomb force then varies as

(6-22)
$$\frac{F_{Quasi\text{-}Coulomb}}{F_{Normal\ Coulomb}} = \frac{{}^{1}/_{n^3}}{{}^{1}/_{n^4}} = n$$

This means that for values of n somewhat larger than that of the next lower stable orbit integer value the actual Coulomb force acting, $F_{Quasi\text{-}Coulomb}$, is too large. For values of n somewhat below the stable orbit integer value the actual Coulomb force acting, $F_{Quasi\text{-}Coulomb}$, is too small.

Those results mean that:

- *Outside or above the stable orbit* integer value of orbit n the excessive values of $F_{Quasi\text{-}Coulomb}$ have the *net effect of moving the electron path inward*. The inward force produces an inward acceleration that is greater than the amount to produce a circular orbit. The excess acceleration produces inward electron velocity. (The inward $F_{Quasi\text{-}Coulomb}$ is greater than the outward "centrifugal force".)

- *Inside or below the stable orbit* integer value of orbit n the insufficient values of $F_{Quasi\text{-}Coulomb}$ have the *net effect of moving the electron path outward*. The inward force produces an inward acceleration that is less than the circular orbit amount. The deficiency produces less than circular motion, a net outward motion effect. (The inward $F_{Quasi\text{-}Coulomb}$ is less than the outward "centrifugal force".)

The overall effect is to force the electrons into stable orbits as Figure 6-3.

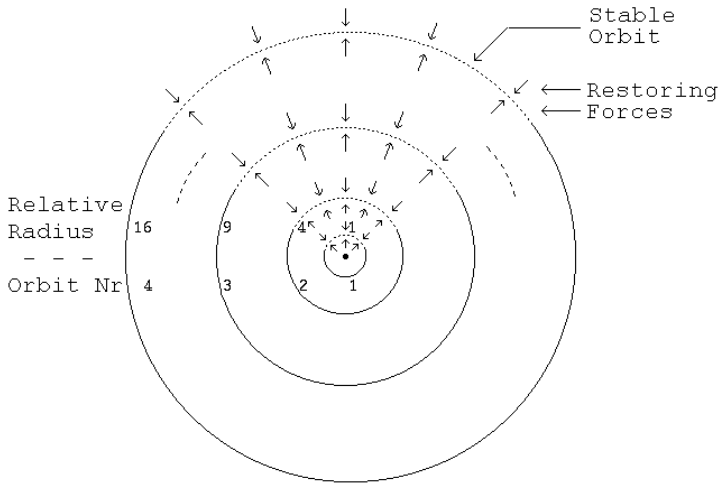

Figure 6-3
The Orbital Electrons Forced Into Integer Matter Wavelength Orbits

The Electrons' Transition Paths Between Stable Orbits

The above Figure 6-3 depicts the status when the orbital electrons are all in their lowest [least energy] orbits. When the outermost of those orbital locations is not occupied and the electron that should be in that position is in an excessively higher orbital location the action of the restoring forces is to direct that electron inward on an orbital transition path to fill the unoccupied position. That happens as follows.

The absence of an electron in the unoccupied position means that the positive electron-attracting field of the atom's positive nucleus is slightly un-offset by the orbital electrons' negative charges. With all of the lowest orbits filled the atom overall presents an electrically neutral status as viewed from outside, but with the outer electron missing that presentation is slightly of inner positive charge as viewed from the excessively higher orbital location electron.

That extra centrally directed attraction curves the pattern of restoring forces of Figure 6-3 to that of Figure 6-4, below.

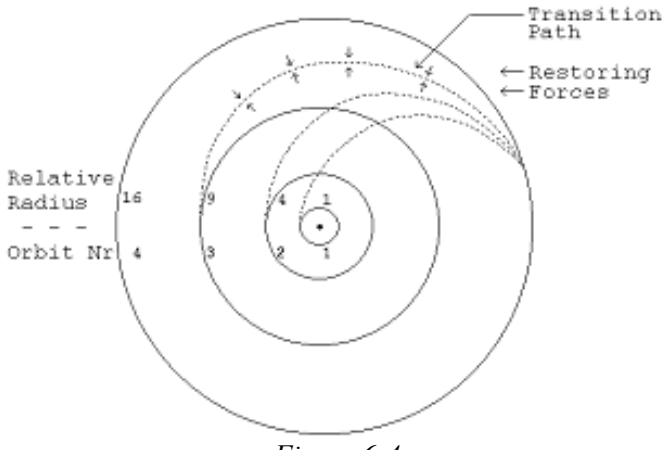

Figure 6-4
The Electrons Orbit Transition Paths

That drives the excessively higher orbital location electron inward to fill the empty location.

Any vacant location in the lowest energy positions of the orbital electron structure is automatically filled from above by this directing of the restoring forces. That is how an outer electron "knows" that there is a space that it can and should move into and that is how it follows the correct path to get there.

From any point in an outer orbit there is one specific path to each of the inner orbits of that outer orbit. Such paths, which involve inward motion of the electron in transition between stable orbits, have at each point in their path the correct inward motion to compensate for the deviation of the value there of $F_{Quasi\text{-}Coulomb}$ from what the normal Coulomb force should be at that point.

The electron velocity must vary smoothly from the stable velocity of the initial outer orbit through a period of increase and ending in the stable velocity of the final orbit. To do that without a discontinuity the variation must be in the form of a half cycle cosine. That is attested to by the sinusoidal nature of the *E-M* radiated photon. There can only be one such path that correctly compensates between any particular pair of initial and final orbits.

On either side of such a path the transition path restoring forces act just as for the stable orbits. The restoring forces arise because the stable orbit restoring forces will not allow locations between stable orbits.

Multi-Electron Atoms' Orbital Electrons Structure

Finally the question arises: what is the allocation of electrons to the stable orbits in multi-electron atoms and what impels the electrons into that structure?

In effect the orbital electron extends a distance of $\frac{1}{2}\cdot\lambda_{mw}$ forward and rearward of its instantaneous location. The space that the matter wave occupies is like a long straight narrow tube tangential to the electron's location on its orbital path.

There are three constraints that govern the behavior of the orbital electrons:

(1) The orbital path length must be an integral number of matter wavelengths, as already developed;

(2) The electrons being all of the same charge magnitude and polarity, tend to repel each other to a spacing equally apart subject to the common central attraction of the oppositely charged nucleus;

(3) The electron spacing along the orbital path must be such that the $\frac{1}{2}\cdot\lambda_{mw}$ extension of the electron in space forward and rearward of its current position does not interfere with the space correspondingly occupied by any of the other electrons.

Of course, in addition there are the obvious constraints that the number of electrons in orbit must be the same as the number of equivalent positive charges in the nucleus because the atom is overall electrically neutral and that the electron orbits and the electron positions in the orbits must be such that they do not collide nor otherwise interfere with each other.

The 20th Century physics model of the orbital electron arrangements is that the electrons are arranged in "shells" [as if in spherical surfaces] designated $n=1$, $n=2$, etc.,

and that there is space for a maximum of: *2* electrons in the *n=1* shell, *8* electrons in the *n=2* shell, *18* electrons in the *n=3* shell, and so on. Those dispositions are correct, but the rules used to determine them, a set of four "Orbital Quantum Numbers", provide no mechanism, no cause for the behavior.

The orbital electron allocation to orbits and arrangement is enforced by the requirement of accommodating the space that each orbiting electron's matter wave occupies, as follows.

Applying the constraints to the innermost *n=1* shell where the orbital path length is $n \cdot \lambda_{mw} = 1 \cdot \lambda_{mw}$ there is only space for *2* electrons in the orbital plane [see Figure 6-4 and equation *(6-23)*, below]. In the figure the second electron is depicted located as close to the first electron as possible without their matter wave extensions in space interfering with each other. Introduction of a third electron into that orbit in that plane would involve spacing that would disrupt the particles and the orbit.

(6-23) For the n = 1 shell the orbital path length, is one wavelength, $2\pi \cdot R = \lambda_{mw}$. Then from Figure 6-4, below:

$$\text{Tan}(\Phi) = \frac{\frac{1}{2} \cdot \lambda_{mw}}{R} = \frac{\frac{1}{2} \cdot 2\pi \cdot R}{R} = \pi$$

$$\Phi = 72.34^{\circ}$$

$$\text{Electron Space} = {}^{360^{\circ}}/_{2 \cdot \Phi} = 2.49 \Rightarrow 2 \text{ electrons}$$

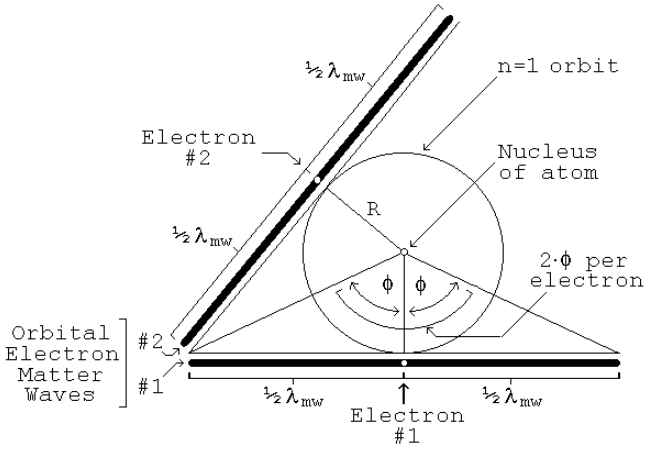

Figure 6-5
Electrons in n=1 Shell

Considering adding more orbital planes, the situation is like a sword dance where a number of dancers whirl and turn, each flashing a pair of swords [matter wave occupied space], while avoiding any casualties among the dancers. The dancers' spacing, paths and timing must be such that while their swords slash at each others' paths they do so only when the dancer in that path and his extended swords is out of the way.

If a plane tilted relative to the above first orbital plane is introduced in the *n=1* shell its first electron will interfere with the prior two regardless of the tilt. Imagining in Figure 6-4, above, that the paper is folded along the line from the nucleus to where the two matter waves just meet the fold tilts one electron's orbital plane relative to the other but does not change the interference of the two. Thus, in terms of the angles in Figure 6-4, a second orbital plane tilted at an angle of $\Phi = 72.34^{\circ}$ or more would seem to fit.

However, the electron in that second orbital plane, starting at $\Phi = 72.34^{\circ}$ above one of the points of intersection with the first plane could travel only the distance $[180^{\circ}-2\cdot\Phi] = 35.32^{\circ}$ before being within $\Phi = 72.34^{\circ}$ of the other side of the orbit, the other point of intersection of the planes. During that 35.32° the pair of electrons in the original plane have not had the necessary travel, $\Phi = 72.34^{\circ}$, to clear their matter wave extensions in space from the common points of intersection of the two orbital planes.

Therefore, the $n=1$ shell can only contain *one orbital plane* with only *one orbit* having *two equally spaced electrons*. Any additional content would involve the matter waves of the electrons interfering with each other.

For the $n=2$ shell the "sword dance" becomes more complex. Clearly, from the above, the first two $n=2$ electrons can readily share an orbit as in the $n=1$ case. In fact, calculation analogous to equation *(6-23)* but for the $n=2$ case shows that three electrons could fit in one $n=2$ orbital plane. That calculation is as follows.

(6-24) For the n = 2 "shell" the orbital path length the circular path circumference, is two matter wavelengths, $2\pi\cdot R = 2\cdot\lambda_{mw}$.

$$\text{Tan}(\Phi) = \frac{\frac{1}{2}\cdot\lambda_{mw}}{R} = \frac{\frac{1}{2}\cdot\pi\cdot R}{R} = \pi/2$$

$$\Phi = 57.52^{\circ}$$

$$\text{Electron Space} = 360^{\circ}/2\cdot\Phi = 3.13 \Rightarrow 3 \text{ electrons}$$

However, the fit is close and more overall equidistant spacing of the electrons is achieved with the third electron occupying a new orbital plane tilted to the first as develops below.

How many such tilted planes can be accommodated at the $n=2$ level in total ? The shell can accommodate three such planes at $\theta = 60^{\circ}$ relative tilts. This limit is set by $\Phi_{n=2} = 57.52^{\circ}$ per equation *6-24*. Four planes tilted at $\theta = 45^{\circ}$ would be too close. The three planes have a common axis of intersection on which are the two points that all three of the orbits have in common (Figure 6-6).

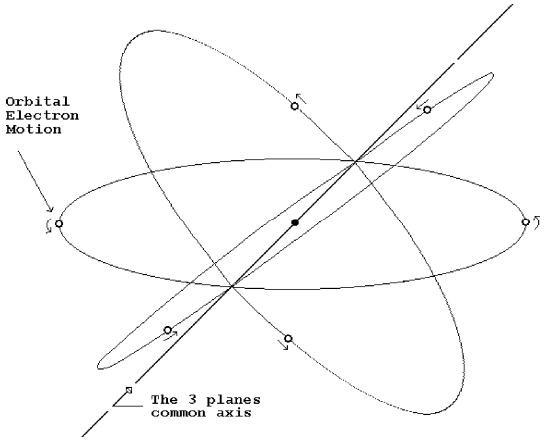

Figure 6-6
Three Orbital Planes and Relative Tilts, n=2 Shell

The six electrons (two per each of three orbital planes tilted at 60° relative to each other) pass through those two common points at $\varphi = 360^{\circ}/6 = 60^{\circ}$ intervals (equidistant spacing). With $\Phi = 57.52^{\circ}$ there is just enough travel between successive electrons for each electron to clear the area before the next one starts arriving.

Can any more electrons fit in this shell ? Yes, two more in another orbital plane perpendicular to the common axis of the other three orbital planes. This new orbit intersects each of the other three successively at $\Theta = 60^{\circ}$ intervals. The two electrons in each such intersected plane are spaced 180° apart. An electron passing such an intersection with one of the first three planes 60° after one of that plane's two electron's has passed and taking 60° to clear the intersection would have cleared the requisite 60° ahead of the other electron of that plane.

Two such electrons 180° apart can be accommodated.

Overall, therefore the $n=2$ shell can fit *eight electrons – two in each of the three common axis planes* plus *two more in the perpendicular plane*.

For $n=3$ the situation becomes considerably more complex. Now the separation angle is $\Phi_{n=3} = 46.32^{\circ}$. The reasoning as for $n=2$, above, indicates that the shell can still accommodate only three orbital planes intersecting on a common axis, each plane having two electrons in orbit 180° apart with the one more plane perpendicular to the common axis of the other three planes. In other words, for $n=3$ the shell appears able to only accommodate the same orbital structure as does the $n=2$ shell. This is in fact the case.

More precisely, the $n=3$ shell so functions until full in that form. Additional electrons for higher Z atoms then start filling the $n=4$ shell. Then, the electric field of those outer $n=4$ electrons becomes sufficient to modify the orbital structure situation and possibilities of the inner $n=3$ shell. The $n=3$ shell then can accommodate the expected five orbital planes on a common axis, each with two electrons, in addition to the already filled $n=2$ type structure.

For higher n the same kind of effect of outer on inner shell modifies the structure, the $n=5$ shell filling partly before the $n=4$ shell is completely filled and that partial outer shell's field then modifying the inner shell's structure.

It is the complex fitting of the space occupied by the orbital electron matter waves into the available integer-matter-wavelength orbital shells that determines the orbital electrons' arrangement structure. That structure is summarized in the table of Figure 6-7, below. The table, arranged so as to directly correspond to the “quantum numbers” system of 20th Century physics shows what those quantum numbers actually represent.

The entire structural effect is the result of the matter waves of the orbital electrons and the restrictions that their space requirements impose on the system.

"Quantum Number"	Orbital Structure
n	The shell's orbital path length is "n" matter wavelengths long. n = 1, 2, 3, …
ℓ	The number of "sets" in a particular "shell" equals $[\ell + 1]$. ℓ = 0, 1, … n–1 A "set" consists of orbital planes of orbits of the same length, tilted at equal angles relative to each other, and sharing the same common axis about which tilted.
m_ℓ	The "index number" of any particular orbital plane in any particular "set" of orbital planes. $m_\ell = +[\ell], +[\ell - 1], \ldots 0, -1, \ldots -[\ell]$ The total number of such orbital planes in the "set" is $[2 \cdot \ell + 1]$, always odd.
m_S	Each individual orbital plane can accommodate 2 electrons equally spaced. m_S = –½ and +½ [for the 1st and 2nd electrons of the plane].

Figure 6-7