Appendix D

Integration Details for Magnetic Effect Calculations

PART (1) -- EQUATION 5 -14, THE STATIC CASE, ↑ COMPONENT

(5-14) $$\uparrow F_E = \int_{-\infty}^{0} \uparrow dF(x) + \int_{0}^{+\infty} \uparrow dF(x)$$

$$= \int_{-\infty}^{0} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx + \int_{0}^{+\infty} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx$$

Since the form of the integral in each of the two regions is the same most of the integration process can be performed on just the form.

(D-1) $$\uparrow \text{Form} = \int_a^b F_r \cdot \frac{R^3}{\left[x^2 + R^2\right]^{1\frac{1}{2}}} \cdot dx$$

$$= F_r \cdot R^3 \cdot \int_a^b \frac{1}{\left[x^2 + R^2\right]^{1\frac{1}{2}}} \cdot dx$$ [F_r and R are constants for this integration]

$$= F_r \cdot R^3 \cdot \frac{x}{R^2 \cdot \left[x^2 + R^2\right]^{\frac{1}{2}}} \Bigg|_a^b$$ [The integration anti-derivative]

[Divide through by $R^2 \cdot x$]

$$= F_r \cdot R \cdot \frac{1}{\left[1 + x^2/R^2\right]^{\frac{1}{2}}} \Bigg|_a^b$$

Returning to the overall equation 14-14 and evaluating at the limits:

(D-2) $$\uparrow F_E = F_r \cdot R \cdot [0 - 1] \quad \text{For}: a = -\infty \quad R^2/\infty^2 = 0$$

$$b = 0 \quad R^2/0^2 = \infty$$

$$= F_r \cdot R \cdot [1 - 0] \quad \text{For}: a = 0 \quad R^2/0^2 = \infty$$

$$b = +\infty \quad R^2/\infty^2 = 0$$

$$= 2 \cdot F_r \cdot R \qquad \text{Overall}$$

PART (2) -- EQUATION 5 -30, CASES 1, 2, & 5, ↑ COMPONENT

(5-30)
$$\uparrow F_T = \int_{-\infty}^{0} \uparrow dF(v,x) + \int_{0}^{+\infty} \uparrow dF(v,x)$$
$$= \int_{-\infty}^{0} \uparrow f(v,x)\cdot dF(x) + \int_{0}^{+\infty} \uparrow f(v,x)\cdot dF(x)$$
$$= \int_{-\infty}^{0} \left[\left[\frac{A\cdot B\cdot(x^2+R^2)}{\left[x^4+(A^2+B^2)\cdot R^2\cdot x^2+A^2\cdot B^2\cdot R^4\right]^{\frac{1}{2}}} + \frac{(C+D)\cdot x}{\left[x^2+R^2\right]^{\frac{1}{2}}}\right]\cdot\left[F_r\cdot\frac{R^3}{\left[x^2+R^2\right]^{1\frac{1}{2}}}\right]\right]\cdot dx$$
$$+\int_{0}^{\infty}\left[\text{The same above expression again for the second integration range}\right]$$

Integrating by parts the following is obtained for $\uparrow F_T$.

(D-3)
$$F_T = \int_{-\infty}^{0} f(v,x)\cdot dF(x) + \int_{0}^{+\infty} f(v,x)\cdot dF(x)$$
$$= \left[f(v,x)\cdot F(x) - \int F(x)\cdot df(v,x)\right]_{-\infty}^{0} + \left[f(v,x)\cdot F(x) - \int F(x)\cdot df(v,x)\right]_{0}^{+\infty}$$

for which f(v,x) and F(x) are as follows.

$$f(v,x) = \left[\frac{A\cdot B\cdot(x^2+R^2)}{\left[x^4+(A^2+B^2)\cdot R^2\cdot x^2+A^2\cdot B^2\cdot R^4\right]^{\frac{1}{2}}} + \frac{(C+D)\cdot x}{\left[x^2+R^2\right]^{\frac{1}{2}}}\right]$$
$$F(x) = F_r\cdot R\cdot\frac{x}{\left[x^2+R^2\right]^{\frac{1}{2}}}$$

The integration from $-\infty$ *to* *0* cancels out with the integration from *0* *to* $+\infty$ in the two integrals. The remaining portion of the integration is the $f(v,x)\cdot F(x)$ portion.

The value of $f(v,x)$ for $x=0$ is *1*. Dividing the numerator and the denominator of the first term of $f(v,x)$ by x^2 and of the second term by x the value of the function for $x = \infty$ is $[A\cdot B+C+D]$. Similarly, $F(x)$ evaluates to $F_r\cdot R\cdot 1$ for $x = \infty$ and to *zero* for $x = 0$.

Therefore the final result is

(D-4)
$$\uparrow F_T = F_r\cdot R\cdot[A\cdot B + C + D]_{>0} + F_r\cdot R\cdot[A\cdot B + C + D]_{<0}$$
$$= 2\cdot F_r\cdot R\cdot[A\cdot B + C + D]$$

$\longrightarrow$