## Appendix B

## The Limitation of the Original Envelopes

This is to show how the otherwise infinite string of envelopes to the original oscillation at the start of the universe was subject to a finite limitation. By "finite limitation" is meant that in the vicinity and beyond of the cut-off number of envelopes, $N_{0}$, the amplitude of each of the further successive envelopes being imposed on the original $U(t)$, equation $2-5$ was successively significantly less than its immediate predecessor and the rate of that amplitude decrease increased sharply with further envelopes - there was a sharp cut-off of amplitude.

The cut-off, as developed below, occurs around the value of $N_{0}$ regardless of what that value is and in that sense the value of $N_{0}$ is not important. Nevertheless it is of interest that various attempts to estimate it give values around 1085 as follows.

## Estimating the Value of no

In the Original $U(t), N_{0}$ is the number of protons and electrons in the Original "Cosmic Egg" and that $N_{0}$, as the exponent of the envelope frequency cosine function, is the effective number of envelopes. The magnitude of that quantity, $N_{0}$, can be approximately determined. The procedure is to calculate the mass of the universe and divide it by the mass of an individual proton, which is

$$
(B-1) \quad \mathrm{m}_{\mathrm{p}}=1.67 \ldots \cdot 10^{-27} \text { kilograms. }
$$

Hydrogen atoms or their equivalent, that is protons and their associated electrons, are the vast majority, more than $99 \%$ of the matter of the universe. The electron is of negligible mass compared to the proton within the limited accuracy of the present calculation, so it is reasonable here to deem the mass of the universe as being all protons for the present purpose.

Determining the mass of the universe, $m_{U}$, proceeds by estimating the average mass density, $\rho$, and the applicable universe volume. The universe mass is then the product of the two and its determination by that procedure is as follows.

Estimates of $\rho$ have been made by estimating the mass of a typical galaxy, that done by estimating the number of stars in a galaxy and multiplying by the estimated average star mass and considering the galaxy's rotational dynamics; then counting the number of galaxies in a volume of space, the process performed for increasingly larger volumes. That procedure has produced a universe mass density estimate of

$$
\rho \approx[\text { from } 1 \text { to } 10] \cdot 10^{-27 ~} \mathrm{~kg} / \mathrm{m}^{3}
$$

Having, then, estimates ranging from about 1 to 10 times 10-27, a reasonable value to use for the mass density of the universe would be the average, about:

$$
(B-2) \quad \rho_{U} \approx 5 \cdot 10-27 \mathrm{~kg} / \mathrm{m}^{3}
$$

Next the applicable volume of the universe is needed so as to obtain the universe's mass as the product of the mass density and the volume. The volume of the universe develops as follows.

The universe's radius applicable to the just obtained universe mass density should be based on an earlier time than the present because the investigations into estimating that density had to treat astral objects which we observe as they were some time in the past -- their distance from us divided by the speed of their light.

Those earlier times were in the range of 0 to 7 or 8 Gyrs into the past. As we look into the past at an increasing radial distance from us the observed volumes increase as that radius cubed. For that reason the applicable universe radius to use with the universe mass density just determined is that which existed at the time into the past $t \approx 6.5$ Gyrs ago.

Estimates of the present radius of the universe are in the range of $10^{27}$ meters. On that basis the radius 6.5 Gyrs ago is estimated to have been 1025 meters.
(B-3) $\quad \mathrm{R}_{\mathrm{U}} \approx 10^{25}$ meters.
Therefore the mass of the universe, as the product of its volume based on that radius and its equation $B-2$ density, is:

$$
(B-4) \quad m_{U}=\rho_{U} \cdot\left[4 / 3 \cdot \pi \cdot R_{U}^{3}\right] \approx 3 \cdot 10^{49} \mathrm{~kg}
$$

and the resulting value of $N_{0}$ is

$$
\begin{aligned}
(B-5) \quad \mathrm{N}_{0} & =\frac{\mathrm{m}_{\mathrm{U}}}{\mathrm{~m}_{\mathrm{p}}} \\
& \approx \frac{3 \cdot 10^{49}}{1.67 \cdot 10^{-27}} \\
& \approx 2 \cdot 1076
\end{aligned}
$$

However, analyses in recent years of the hypothesized or speculated likely scenario of the early universe, the "big bang", result in the rough estimate that there were then about $10^{9}$, one billion, mutual annihilations for every proton present today. (This is based upon the observation that in the present day universe there are about 109 photons per proton. That estimate is a not unreasonable measure of the original number of annihilations. The mutual annihilations each produced two photons. Photons from other later causes, primarily black body radiation and electron orbital changes should be in an amount on the order of one photon per proton, far from the 109 , and leaving the Original mutual annihilations to account for that).

In that case the $2 \cdot 1076$ estimate for the present number of particles would give an Original $N_{0}$ value, at the initial instant before any mutual annihilations, of about $2 \cdot 1085$. While all of this estimating is quite approximate it would nevertheless be fairly reasonable to take that $N_{0}$ was on the order of 1085 .

## the Limitation of the Original Envelopes

After a moderate number of such cut-off region envelopes the amplitude of any further envelopes becomes infinitesimal. While such infinitesimal (and still continuing to become ever more infinitesimal) envelopes theoretically go on to an infinite number of them, the result is equivalent to the convergence to a finite value of a mathematical infinite series such as, for example that of the cosine. The envelopes cut-off is a result of the mathematics of $U(t)$.

The key to that behavior is to be found in Table B-1, below, the expansion of the $\operatorname{Cos}^{n}(x)$ function. The "Cosmic Egg" expression, Section 2, equation 2-5, repeated below

$$
(2-5) \quad \mathrm{U}(\mathrm{t})= \pm \mathrm{U}_{0} \cdot\left[1-\operatorname{Cos}\left[2 \cdot \pi \cdot \mathrm{f}_{\text {env }} \cdot \mathrm{t}\right]\right]^{\mathrm{N}_{0}} \cdot\left[1-\operatorname{Cos}\left[2 \cdot \pi \cdot \mathrm{f}_{\text {wve }} \cdot \mathrm{t}\right]\right]
$$

contains the factor
(B-6)

$$
\cos ^{N_{0}}\left[2 \pi\left(f_{\text {env }}\right) t\right]
$$

which creates the set of envelopes to the original oscillation. The expansion of the cosine raised to the power of its $N_{0}$ exponent behaves according to the pattern illustrated in Table B-1, below. Analysis of the patterns in the coefficients of the individual terms of the $\operatorname{Cos}^{n}(x)$ expansion discloses a pattern related to the binomial expansion as demonstrated in the table.
(a) Binomial Expansion Coefficients $[a+b]^{n}$

(b) $\operatorname{Cos}^{n}(x)$ Expansion Coefficients


## Table B-1

Clearly, with the exception of the constant term (where, in the table, ${ }^{*}=0 x$ ) the other terms of the expansion of $\cos ^{n}(x)$ have the same coefficients as the corresponding terms of the binomial expansion. The formula for the binomial expansion can thus be used to obtain the coefficients for any value of $n$ in the expansion of $\operatorname{Cos}^{n}(x)$. in the present case for any value of $N_{0}$ in the expansion of the $U(t)$ factor $\operatorname{Cos}^{N}{ }^{N}\left[2 \pi\left(f_{\text {env }}\right) t\right]$
$N_{0}=1085$ is the $n$ of the formula. It is not practicable and most likely not possible to calculate all of the coefficients of the cosine expansion of the envelopes for 1085 envelopes. On the other hand, it is not unreasonable to calculate the 85 cases corresponding to the frequency multiples of the expansion: $101,102,103, \ldots$ 1085.

Figure B-1, below, is a plot of the relative magnitude of the successive coefficients of the various frequency multiples (1•x, $3 \cdot x, \cdots 1085 \cdot x)$, in the expansion of $\operatorname{Cos}^{n}(x)$ for $n=N_{0}=1085$. The plot indicates a sharp cut-off, an attenuation of the higher frequencies. Figure B-1(a) uses a linear horizontal axis and shows the cut-off in detail. Figure B-1(b) uses a logarithmic horizontal scale to better present the tremendous range in frequency multiples from 1 to 1085. It shows that the cut-off is quite sharp and drastic.

This cut-off is merely the action of the mathematics of $\cos ^{n}(x)$.

(a) Linear Scale

Figure B-2
The $\operatorname{Cos}^{n}(x)$ Limitation of the "Cosmic Egg

(b) Logarithmic Scale

Figure B-2
The $\operatorname{Cos}^{n}(x)$ Limitation of the "Cosmic Egg

